

# Unemployment Fluctuations with Staggered Nash Wage Bargaining

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A number of authors have argued that the conventional model of unemployment dynamics due to Mortensen and Pissarides has difficulty accounting for the relatively volatile behavior of labor market activity over the business cycle. We address this issue by modifying the Mortensen-Pissarides framework to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. We then show that a reasonable calibration of the model can account for the cyclical behavior of wages and labor market activity observed in the data.

## I. Introduction

A long-standing challenge in macroeconomics is accounting for the relatively smooth behavior of real wages over the business cycle along with the relatively volatile behavior of employment. A recent body of research, beginning with Hall (2005*b*), Shimer (2005), and Costain and Reiter (2008), has reignited interest in addressing this challenge. These

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authors show that the conventional model of unemployment dynamics due to Mortensen and Pissarides cannot account for the key cyclical movements in labor market activity, at least for standard calibrations of parameters. The basic problem is that the mechanism for wage determination within this framework, period-by-period Nash bargaining between firms and workers, induces too much volatility in wages. This exaggerated procyclical movement in wages, in turn, dampens the cyclical movement in firms' incentives to hire. Shimer (2004) and Hall (2005*b*) proceed to show that with the introduction of ad hoc wage stickiness, the framework can account for employment volatility. Of course, this raises the question of what the primitive forces are that might underlie this wage rigidity.

A rapidly growing literature has emerged to take on this puzzle. Much of this work attempts to provide an axiomatic foundation for wage rigidity, explicitly building up from assumptions about the information structure, and so on.<sup>1</sup> To date, because of complexity, this work has focused mainly on qualitative findings and has addressed quantitative issues only in a limited way.<sup>2</sup>

In this paper we take a pragmatic approach to modeling wage rigidity, with the aim of developing a framework that is tractable for quantitative analysis. In particular, we retain the intuitively appealing feature of Nash bargaining but modify the conventional Mortensen-Pissarides model to allow for staggered multiperiod wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is made between a firm and its existing workforce. Assuming that all employees of the same productivity receive the same wage, workers hired in between contract settlements receive the existing contract wage. We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. The gain over a simple ad hoc wage adjustment mechanism is that the key primitive parameter of the model is the average frequency of wage adjustment, as opposed to an arbitrary partial adjustment coefficient in a wage equation. In this way, the staggered contracting structure pro-

<sup>1</sup> Examples include Shimer and Wright (2004), Menzio (2005), Kennan (2006), and Hall and Milgrom (2008). Others have explored flexible wage alternatives, e.g., Krause and Lubik (2007), Mortensen and Nagypal (2007), Hagedorn and Manovskii (2008), and Rotemberg (2008). See Hall (2005*c*) for a survey.

<sup>2</sup> An exception is the paper by Menzio (2005), who presents a calibrated model with endogenous wage rigidity. His model does well except for wages, which are too smooth. We instead focus on explaining the joint dynamics of labor market activity and wages.

vides more discipline in evaluating the model than simple ad hoc adjustment mechanisms do.

The use of time-dependent staggered price and wage setting, of course, is widespread in macroeconomic modeling, beginning with Taylor (1980) and Calvo (1983). More recently, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) have found that staggered wage contracting is critical to the empirical performance of the recent vintage of dynamic general equilibrium macroeconomic frameworks (i.e., sticky prices alone are not sufficient). There are, however, some important distinguishing features of our approach. First, macroeconomic models with staggered wage setting typically have employment adjusting along the intensive margin. That is, wage stickiness enhances fluctuations in hours worked as opposed to total employment. As a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firms and workers have an ongoing relationship. If wages are not allocational, of course, then wage rigidity does not influence model dynamics. By contrast, in the model we present, wages affect employment at the extensive margin: They influence the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005*b*), in this kind of setting Barro's critique does not apply.

A second key difference involves the nature of the wage contracting process. In the conventional macroeconomic models, monopolistically competitive workers set wages. Here, firms and workers bargain over wages in a setting with search and matching frictions. As a consequence, some interesting "spillover" effects emerge of the average market wage on the contract wage. These spillover effects are a product of the staggered contract/bargaining environment. They introduce additional stickiness in the movement of real wages, much the same way that real rigidities enhance nominal price stickiness in models of staggered price setting (e.g., Kimball 1995; Woodford 2003).

In Section II we present the basic ingredients of the model, as well as the definition of the equilibrium. In Section III, to develop some intuition about how the model works, we characterize the steady state and then derive a set of simple dynamic equations for wages and hiring, obtained by considering a local approximation of the model about the steady state. We also exposit the spillover effects that influence the wage bargaining process, contributing to overall wage stickiness. One additional distinguishing feature of the setup is that a "horizon effect" emerges that influences the bargaining process, since firms care about the implications of the contract wage for future hires but workers do not. While the horizon effect is interesting from a theoretical perspective, it turns out not to be quantitatively important in our baseline calibration. In Section IV we examine the empirical performance of the

model and show that the framework does a good job of accounting for the basic features of the U.S. data, including wage dynamics.

In Section V, we address three key issues concerning the robustness of our analysis. First, we show that under our baseline calibration, wages almost surely lie within the bargaining set over the life of the contract, which justifies our solution to both the firm's decision problem and the overall bargaining problem. Second, we show that the losses to a firm and its workers from opting for multiperiod as opposed to period-by-period contracts are quite small, which provides some support for our contract structure. Finally, Pissarides (2007) and Haefke, Sonntag, and Van Rens (2008) have recently argued that the evidence on the relative cyclicality of wages of new hires is inconsistent with our assumption that, in between contracts, the wages of new hires are tied to those of similarly productive existing workers. We argue that this is not the case and present some new evidence in support of our contention.

Concluding remarks are in Section VI. Finally, Appendix A discusses the nonstandard features that multiperiod contracting introduces in our Nash bargaining problem. Appendix B presents the complete log-linearized model.<sup>3</sup>

## II. The Model

The framework is a variation of the Mortensen and Pissarides search and matching model (Mortensen and Pissarides 1994; Pissarides 2000). The main difference is that we allow for staggered multiperiod wage contracting. Within the standard framework, workers and firms negotiate wages on the basis of period-by-period Nash bargaining. We keep the Nash bargaining framework, but in the spirit of Taylor (1980) and Calvo (1983), only a fraction of firms and workers reset wages in any given period. Also, they strike a bargain that lasts for multiple periods. We assume that workers hired in between contracting periods receive the contract wage for existing workers of similar productivity. Here the idea is that owing to scale economies in bargaining, employment terms are negotiated only periodically: Firms do not negotiate separate terms for the relatively small percentage of workers who enter in between contracting periods. In the spirit of Hall (2005*b*), the contract wage provides the wage norm for newly hired workers. Though we defer to Section V a discussion of the formal evidence on this issue, we note in the meantime that survey evidence from both the United States (Bewley 1999) and the euro area (Galuscak et al. 2008) supports this proposition.

<sup>3</sup> The extended appendix in the December 2008 working paper version of this paper, available online at the authors' Web sites, provides an explicit derivation of all the key results, including the steady-state model.

For technical reasons, there are two other differences from the Mortensen-Pissarides model. First, because it will turn out to be important for us to distinguish between existing and newly hired workers at a firm, we drop the assumption of one worker per firm and instead allow firms to hire a continuum of workers. We assume constant returns to scale, however, which greatly simplifies the bargaining problem. Second, we drop the conventional assumption of a fixed cost per vacancy opened and instead assume that firms face quadratic adjustment costs of adjusting employment size.<sup>4</sup> The reason is as follows: With staggered wage setting, there will arise a dispersion of wages across firms in equilibrium. Quadratic costs of adjusting employment ensures a determinate equilibrium in the presence of wage dispersion.

We embed our search and matching framework within a simple intertemporal general equilibrium framework in order to study the dynamics of unemployment and wages. Following Merz (1995) and others, we adopt the representative family construct, which effectively involves introducing complete consumption insurance.

Finally, since the environment is stationary, we omit the use of time subscripts and denote the current value of any variable  $y$  with  $y$  and the next-period value with  $y'$ .

#### A. *Unemployment, Vacancies, and Matching*

Let us now be more precise about the details: There is a continuum of infinitely lived workers and a continuum of infinitely lived firms, each of measure one. Each firm employs  $n$  workers in the current period. It also posts  $v$  vacancies in order to attract new workers for the next period of operation. The total numbers of vacancies and employed workers are  $\bar{v} = \int_i v di$  and  $\bar{n} = \int_i n di$ , where firms are indexed by  $i$ . The total number of unemployed workers searching for a job is

$$\bar{u} = 1 - \bar{n}.$$

Following convention, we assume that the aggregate number of new hires or “matches,”  $\bar{m}$ , is a function of unemployed workers and vacancies, as follows:

$$\bar{m} = \sigma_m \bar{u}^\sigma \bar{v}^{1-\sigma}, \quad (1)$$

<sup>4</sup> We allow for quadratic costs of adjusting employment as opposed to vacancies to keep the algebra as simple as possible. The main cyclical properties of the model are preserved under either approach.

where the parameter  $\sigma_m$  reflects the efficiency of the matching process. The current probability a firm fills a vacancy,  $q$ , is given by

$$q = \frac{\bar{m}}{\bar{v}}. \quad (2)$$

Similarly, the probability an unemployed worker finds a job,  $p$ , is given by

$$p = \frac{\bar{m}}{\bar{u}}. \quad (3)$$

Both firms and workers take  $q$  and  $p$  as given.

Finally, each firm exogenously separates from a fraction  $1 - \rho$  of its workers each period, where  $\rho$  is the probability a worker “survives” with the firm until the next period. Accordingly, within our framework, fluctuations in unemployment will be due to cyclical variation in hiring as opposed to separations. Both Hall (2005a, 2005c) and Shimer (2005, 2007) argue that this characterization is consistent with recent U.S. evidence.

### B. Firms

Every period, each firm produces output,  $y$ , using capital,  $k$ , and labor,  $n$ , according to the following Cobb-Douglas technology:

$$y = zk^\alpha n^{1-\alpha}, \quad (4)$$

where  $z$  is a common productivity factor that is stochastic and follows a first-order Markov process. As we noted earlier, because we will have wage dispersion across firms, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs. For simplicity, we assume that capital is perfectly mobile across firms and that there is a competitive rental market in capital.

It is convenient to define the hiring rate,  $x$ , as the ratio of new hires,  $qv$ , to the existing workforce,  $n$ :

$$x = \frac{qv}{n}. \quad (5)$$

Note that the firm knows the current hiring rate with certainty, since it knows the likelihood  $q$  that each vacancy it posts will be filled. Next period, in turn, total workforce is the sum of the number of surviving workers,  $\rho n$ , and new hires,  $xn$ :

$$n' = (\rho + x)n. \quad (6)$$

Let  $\mathbf{s}$  be the set of variables that define the aggregate state; let

$\beta\Lambda(\mathbf{s}, \mathbf{s}')$  be the firm's discount rate, where the parameter  $\beta$  is the household's subjective discount factor and  $\Lambda(\mathbf{s}, \mathbf{s}')$  is defined below. Let  $r$  be the rental rate of capital, and let  $w$  be the firm's wage rate. Assume that the costs of adjusting the workforce are quadratic and proportional to the existing workforce size. Then, conditional on the current wage and employment, the value  $F$  of each firm may be expressed as

$$F(n, w, \mathbf{s}) = \max_{k,x} \left\{ zk^\alpha n^{1-\alpha} - wn - \frac{\kappa}{2} x^2 n - rk \right. \\ \left. + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')F(n', w', \mathbf{s}')|n, w, \mathbf{s}\} \right\} \quad (7)$$

subject to equation (6). For the time being we take the firm's expected wage path as given, but then in Section II.D we describe the wage determination process.

With constant returns to scale with perfectly mobile capital, the marginal product of a worker equals the average product. The average product in turn is independent of the employment size of the firm. Given this homogeneity in the firms' technology, we argue that it is reasonable to restrict attention to a "fundamentals-based" solution for the path of wages, where the outcome of the wage bargain is similarly independent of the firm's scale.<sup>5</sup> It then follows that the value of the firm conditional on the current wage is linear in its current employment level, as follows:

$$F(n, w, \mathbf{s}) = J(w, \mathbf{s})n, \quad (8)$$

where, with  $\check{k} = k/n$  denoting the firm's capital/employment ratio,

$$J(w, \mathbf{s}) = \max_{\check{k},x} \left\{ z\check{k}^\alpha - w - \frac{\kappa}{2} x^2 - r\check{k} \right. \\ \left. + (\rho + x)\beta E\{\Lambda(\mathbf{s}, \mathbf{s}')J(w', \mathbf{s}')|w, \mathbf{s}\} \right\}. \quad (9)$$

At any time, the firm maximizes its value by choosing the hiring rate

<sup>5</sup> In Sec. II.D, we describe the wage bargain in detail. Given our assumptions on technology, the outcome of the current wage bargain is indeed independent of the firm's scale, as long as the relevant parties believe that the same is true of the outcome of future wage bargains. We have not explored whether there may be self-fulfilling solutions in which firm scale matters to current wages simply because the parties believe that it will matter in the future. We believe that our "fundamentals-based" solution is the most natural starting point, given the environment, and defer to future research whether or not there may be interesting sunspot equilibria to consider.

(by posting vacancies) and its capital/employment ratio, given the probability of filling a vacancy, the rental rate on capital, and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well as the likelihood it will be renegotiating in the future. We next consider the firm's hiring and capital rental decisions and defer a bit the description of the wage bargain.

The first-order condition for the capital/employment ratio is simply

$$r = \alpha \check{k}^{\alpha-1}, \quad (10)$$

where, owing to Cobb-Douglas production and perfectly mobile capital,  $\check{k}$  is the same across firms.

The first-order condition for hiring equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa x = \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')J(w', \mathbf{s}')|w, \mathbf{s}\}. \quad (11)$$

Combining equations yields the following forward-looking difference equation for the hiring rate:

$$\kappa x = \beta E\left\{\Lambda(\mathbf{s}, \mathbf{s}')\left(a' - w' + \frac{\kappa}{2}x'^2 + \rho\kappa x'\right)\right|w, \mathbf{s}\}, \quad (12)$$

where  $a$  denotes the current marginal product of labor (i.e.,  $a = [1 - \alpha]y/n$ ), which is independent of the firm. The hiring rate thus depends on a discounted stream of the firm's expected future surpluses from the marginal worker: the sum of net earnings at the margin,  $a' - w'$ , and saving on adjustment costs,  $(\kappa/2)x'^2$ .

Observe that the only firm-specific variable the hiring rate  $x$  depends on is the wage  $w$ . Thus, all firms with the same wage  $w$  choose the same hiring rate  $x$ , regardless of their respective employment size.

### C. Workers

In this subsection we develop an expression for a worker's surplus from employment, which becomes a critical determinant of the outcome of the wage bargain. Given constant returns to scale, a worker's welfare is independent of the employment size of the firm at which he is working. Given this consideration and given that we effectively have complete consumption insurance, we may index workers simply by their respective current wage,  $w$ . That is, all workers with wage  $w$  will have the same welfare, regardless of which firm they are employed at.

Accordingly, we can express the current value of employment (in

consumption good equivalents) as  $V(w, \mathbf{s})$ . Let  $U(\mathbf{s})$  be the current value of unemployment. Then  $V(w, \mathbf{s})$  is given by

$$V(w, \mathbf{s}) = w + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')[\rho V(w', \mathbf{s}') + (1 - \rho)U(\mathbf{s}')]|w, \mathbf{s}\}. \quad (13)$$

Note that this value depends on the worker's wage,  $w$ , as well as the likelihood the worker will remain employed in the subsequent period.

To construct the value of unemployment, we first define  $\bar{V}_x(\mathbf{s})$  as the average value of employment conditional on being a new worker in the current period. The subscript  $x$  is meant to denote that we are averaging  $V(w, \mathbf{s})$  across new workers, that is, workers who were hired in the previous period. Let  $G(w, \mathbf{s})$  be the cumulative distribution function of wages in state  $\mathbf{s}$ , that is, the fraction of workers who have a wage less than or equal to  $w$ , given the macro state  $\mathbf{s}$ . Then  $\bar{V}_x(\mathbf{s}')$  is given by

$$\bar{V}_x(\mathbf{s}') = \int_w V(w', \mathbf{s}') \frac{x(w, \mathbf{s})}{\bar{x}} dG(w, \mathbf{s}), \quad (14)$$

where  $x(w, \mathbf{s})$  is the hiring rate at firms with wage  $w$  in state  $\mathbf{s}$  and  $\bar{x}$  is the average hiring rate, given by<sup>6</sup>

$$\bar{x} = \int_w x(w, \mathbf{s}) dG(w, \mathbf{s}). \quad (15)$$

Next, let  $b$  be the flow value from unemployment, taken to be unemployment benefits. Then  $U(\mathbf{s})$  may be expressed as

$$U(\mathbf{s}) = b + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')[p\bar{V}_x(\mathbf{s}') + (1 - p)U(\mathbf{s}')]|s\}, \quad (16)$$

where, as before,  $p$  is the probability of finding a job for the subsequent period. The value of unemployment thus depends on the current flow value  $b$  and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker who is currently unemployed is  $\bar{V}_x(\mathbf{s}')$ , the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.<sup>7</sup>

<sup>6</sup> Thus  $\bar{V}_x(\mathbf{s}')$  is distinct from the unconditional average value of employment:  $\bar{V}(\mathbf{s}') = \int_w V(w', \mathbf{s}') dG(w', \mathbf{s}')$ . However, since in the steady state wages and hiring rates are identical across firms,  $\bar{V}_x(\mathbf{s}')$  and  $\bar{V}(\mathbf{s}')$  are identical in the steady state and have similar dynamics outside the steady state, up to a first-order approximation. Key to the latter result is that in the steady state the distribution of wages is degenerate.

<sup>7</sup> There is accordingly no directed search. Note, however, that wage differentials across firms are due only to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

The worker surplus at a firm paying a wage  $w$ ,  $H(w, \mathbf{s})$ , and the average worker surplus conditional on being a new hire,  $\bar{H}_x(\mathbf{s})$ , are given by

$$H(w, \mathbf{s}) = V(w, \mathbf{s}) - U(\mathbf{s}) \quad (17)$$

and

$$\bar{H}_x(\mathbf{s}) = \bar{V}_x(\mathbf{s}) - U(\mathbf{s}). \quad (18)$$

It follows that

$$H(w, \mathbf{s}) = w - b + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')[\rho H(w', \mathbf{s}') - p\bar{H}_x(\mathbf{s}')]|w, \mathbf{s}\}. \quad (19)$$

#### D. Nash Bargaining and Wage Dynamics

As we noted in the introduction, we restrict the form of the wage contract to call for a fixed wage per period over an exogenously given length of time. Given these restrictions on the form of the contract, workers and firms use a Nash bargaining approach to determine the contract wage.

We introduce staggered multiperiod wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability  $1 - \lambda$  that it may renegotiate the wage. This adjustment probability is independent of its history. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at  $1/(1 - \lambda)$ . The coefficient  $\lambda$  is thus a measure of the degree of wage stickiness that can be calibrated to match the data.<sup>8</sup> This simple Poisson adjustment process further implies that it is not necessary to keep track of individual firms' wage histories, which makes aggregation simple. In the end, the model will deliver a simple log-linear relation for the evolution of wages that is the product of Nash bargaining in conjunction with staggered wage setting.

Firms that enter a new wage agreement negotiate with the existing workforce, including the recent new hires. Owing to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly negotiated wage.<sup>9</sup> When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the wage paid the previous

<sup>8</sup> This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

<sup>9</sup> To be clear, with constant returns, one could think of the firm either bargaining with each marginal worker individually or bargaining with a union that wishes to maximize average worker surplus.

period. As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for workers who arrive in between contracting periods and instead, in Hall's terminology, have the existing contract wage provide the wage norm for these new employees. Of course, the newly hired workers recognize that they will be able to renegotiate the wage at the next round of contracting.

Before proceeding, we note that the multiperiod contracting introduces a nonstandard element to our Nash bargaining problem. In contrast to the conventional setup, the wage negotiated for the current period affects the hiring rate since there is a probability  $\lambda$  that the contract will continue the next period. As we show in Appendix A, this makes the bargaining set nonconvex, unlike the standard Nash problem. However, the departure from convexity is not quantitatively large. Thus, as we show in Appendix A, under our calibration, the first-order conditions define a global optimum. It is true that the nonconvex bargaining set opens up the possibility that a wage lottery may be preferred. However, as we show in the extended appendix of the working paper version of this paper, under reasonable conditions the gains from the lottery are small and easily offset by small transactions costs of running and enforcing the lottery. Thus in what follows we restrict attention to deterministic wage contracts.

Let  $w^*$  denote the wage of a firm that renegotiates in the current period. As just noted, the firm negotiates with the marginal worker over the surplus from the marginal match. The bargaining problem is to choose the contract wage  $w^*$  to maximize the Nash product, given by

$$H(w, \mathbf{s})^\eta J(w, \mathbf{s})^{1-\eta} \quad (20)$$

subject to

$$w' = \begin{cases} w & \text{with probability } \lambda \\ w^{*'} & \text{with probability } 1 - \lambda, \end{cases} \quad (21)$$

where  $w^{*'}$  is the wage in the subsequent period if the firm is able to renegotiate and  $J(w, \mathbf{s})$  and  $H(w, \mathbf{s})$  are defined as in equations (9) and (19), respectively. We emphasize that  $w^{*'}$  is chosen optimally at  $t + 1$  and is thus independent of  $w$ .

The first-order necessary condition for the Nash bargaining problem is given by

$$\eta \epsilon(\mathbf{s}) J(w^*, \mathbf{s}) = (1 - \eta) \mu(w^*, \mathbf{s}) H(w^*, \mathbf{s}), \quad (22)$$

where  $\epsilon(\mathbf{s}) \equiv \partial H(w, \mathbf{s}) / \partial w$  is the effect of a rise in the contract wage on the worker's surplus,  $\mu(w, \mathbf{s}) \equiv -\partial J(w, \mathbf{s}) / \partial w$  is minus the effect of a rise in the contract wage on the firm's surplus,

$$\epsilon(\mathbf{s}) = 1 + (\rho\lambda\beta) E\{\Lambda(\mathbf{s}, \mathbf{s}') \epsilon(\mathbf{s}') | \mathbf{s}\}, \quad (23)$$

and<sup>10</sup>

$$\mu(w, \mathbf{s}) = 1 + [\rho + x(w, \mathbf{s})](\lambda\beta)E\{\Lambda(\mathbf{s}, \mathbf{s}')\mu(w, \mathbf{s}')|\mathbf{s}\}. \quad (24)$$

Observe that  $\epsilon(\mathbf{s})$  is effectively the cumulative discount factor the worker uses to value the contract wage stream, and  $\mu(w, \mathbf{s})$  is that for the firm. Since the hiring rate  $x(w, \mathbf{s})$  must be nonnegative,  $\mu(w, \mathbf{s}) \geq \epsilon(\mathbf{s})$ , implying that the firm places a greater weight on the future than the worker does. Intuitively, the firm has a longer horizon than the worker because it cares about the effect of the current wage contract on payments not only to the existing workforce but also to new workers who enter under the terms of the existing contract. A worker, however, cares about wages only during his or her tenure at the firm.

It is possible to rewrite equation (22) as

$$\chi(w^*, \mathbf{s})J(w^*, \mathbf{s}) = [1 - \chi(w^*, \mathbf{s})]H(w^*, \mathbf{s}), \quad (25)$$

with

$$\chi(w, \mathbf{s}) = \frac{\eta}{\eta + (1 - \eta)\mu(w, \mathbf{s})/\epsilon(\mathbf{s})}. \quad (26)$$

Equation (26) is a variation on the conventional sharing rule, where the relative weight  $\chi$  depends not only on the worker's bargaining power  $\eta$  but also on the differential firm/worker horizon, reflected by the term  $\mu/\epsilon$ . Note that in the limiting case of  $\lambda = 0$ ,  $\mu/\epsilon = 1$  and  $\chi = \eta$ , as in the conventional case of period-by-period wage bargaining. With  $\lambda > 0$ , however,  $\chi$  is less than  $\eta$  (since  $\mu/\epsilon$  exceeds unity). In analogy to Binmore, Rubinstein, and Wolinsky (1986), because the horizon effect effectively makes the worker more impatient than the firm, it effectively reduces the worker's bargaining power from  $\eta$  to  $\chi$ .

It is also straightforward to show that in the limiting case of  $\lambda = 0$ , the wage that solves the bargaining problem is given by

$$w_{\chi=0}^* = \eta \left[ a + \frac{\kappa}{2} x(w^*, \mathbf{s})^2 \right] + (1 - \eta)[b + p\beta E\{\Lambda(\mathbf{s}, \mathbf{s}')\bar{H}_x(\mathbf{s}')|\mathbf{s}\}]. \quad (27)$$

As in the conventional formulation, with period-by-period Nash bargaining, the wage is a convex combination of what a worker contributes to the match (the first term on the right) and what the worker loses by accepting a job (the second term), where the weights depend on the

<sup>10</sup> As App. A shows, the dependency of  $\mu(w, \mathbf{s})$  on  $w$  (via the effect of  $w$  on  $x(w, \mathbf{s})$ ) is the source of the nonconvexity in the bargaining set. However, because this effect is not quantitatively large, the Nash first-order conditions still define a global optimum over the space of deterministic wage contracts.

relative bargaining power.<sup>11</sup> With multiperiod contracting, however, the contract wage will also depend on expected future conditions. As we show in Section III.B, with  $\lambda > 0$ , up to a first order,  $w^*$  equals an expected discounted sum of current and expected future values of a target wage  $w^o$ . The target wage  $w^o$ , further, is the wage that would arise under period-by-period Nash bargaining, modified to allow for the horizon effect.

We next characterize the relation between the contract wage  $w^*$  and the evolution of the average wage  $\bar{w}$  across workers, given by

$$\bar{w} = \int_w w dG(w, \mathbf{s}). \quad (28)$$

Since the fraction of firms and workers that renegotiate contracts is a random draw across the population and since all firms and workers that renegotiate choose the same contract wage, by the law of large numbers we can express the wage index recursively as

$$\bar{w}' = (1 - \lambda)w^{*'} + \lambda \int_w w \frac{\rho + x(w, \mathbf{s})}{\rho + \bar{x}} dG(w, \mathbf{s}), \quad (29)$$

where  $1 - \lambda$  is the fraction of workers who are renegotiating and  $\lambda$  is the fraction who are not. The next-period average wage,  $\bar{w}'$ , is thus a convex combination of the next-period contract wage,  $w^{*'}$ , and the average wage across the population of workers who do not renegotiate, given by the integral in the second term. As we show below, up to a first order, this integral can be approximated by the current-period average wage,  $\bar{w}$ .<sup>12</sup>

Finally, given the process for the evolution of wages, the fraction of workers with wage  $w$  evolves according to

$$dG(w, \mathbf{s}') = \begin{cases} \lambda \frac{\rho + x(w, \mathbf{s})}{\rho + \bar{x}} dG(w, \mathbf{s}) & \forall w \neq w^{*'} \\ \lambda \frac{\rho + x(w, \mathbf{s})}{\rho + \bar{x}} dG(w, \mathbf{s}) + (1 - \lambda) & \text{if } w = w^{*'}, \end{cases} \quad (30)$$

where  $dG(w, \mathbf{s})$  is the fraction of workers with wage  $w$  in state  $\mathbf{s}$  and  $dG(w, \mathbf{s}')$  is the corresponding fraction in state  $\mathbf{s}'$ . Given the staggered

<sup>11</sup> One small difference from the standard formulation is that because of our quadratic adjustment cost formulation, the saving on adjustment costs that is a component of the worker's value to the match is given by  $(\kappa/2)x^2$ .

<sup>12</sup> The integral  $\int_w w \{[\rho + x(w, \mathbf{s})]/(\rho + \bar{x})\} dG(w, \mathbf{s})$  is not quite identical to  $\bar{w} = \int_w w dG(w, \mathbf{s})$ . However, given that the hiring rates are identical across firms in the steady state, the dynamics of this integral are equivalent to the dynamics of  $\bar{w}$ , up to a first order. Key to the result is that the steady-state wage distribution is degenerate as is the hiring rate.

wage contracting, the aggregate productivity shock creates dispersion of wages across workers. It is straightforward to show, however, that in the deterministic steady state, all workers receive the target wage and in the absence of new shocks the dispersion of wages converges to this degenerate distribution.

### E. Consumption and Saving

Following Merz (1995) and others, we use the representative family construct, which gives rise to perfect consumption insurance. In particular, the family has a continuum of employed workers at all firms and unemployed workers, representative of the population at large. The family pools their incomes before choosing per capita consumption and asset holdings. In addition to wage income and unemployment income, the family has a diversified ownership stake in firms, which pays out profits. Finally, households may either consume  $\bar{c}$  or save in the form of capital  $\bar{k}$ , which they rent to firms at the rate  $r$ . Let  $\Omega(\mathbf{s})$  be the value function for the representative household. Then the maximization problem may be expressed as

$$\Omega(\mathbf{s}) = \max_{\bar{c}, \bar{k}'} \{ \log(\bar{c}) + \beta E\{\Omega(\mathbf{s}') | \mathbf{s}\} \} \quad (31)$$

subject to

$$\bar{c} + \bar{k}' = \bar{w}\bar{n} + (1 - \bar{n})b + (1 - \delta + r)\bar{k} + T + \Pi, \quad (32)$$

where  $T$  is lump-sum transfers from the government and  $\Pi$  is profits received from firms, taken as given by the households.

Given the law of large numbers, total household employment  $\bar{n}$  evolves according to

$$\bar{n}' = (\rho + \bar{x})\bar{n}, \quad (33)$$

where the family takes the average hiring rate  $\bar{x}$  as given.

Then the first-order necessary condition for consumption/saving yields

$$1 = (1 - \delta + r)\beta E\{\Lambda(\mathbf{s}, \mathbf{s}') | \mathbf{s}\}, \quad (34)$$

where  $\Lambda(\mathbf{s}, \mathbf{s}') = \bar{c}/\bar{c}'$ .

*F. Resource Constraint, Government Policy, and Equilibrium*

We complete the model with the following resource constraint, which divides output between consumption, investment, and adjustment costs:

$$\bar{y} = \bar{c} + \bar{k}' - (1 - \delta)\bar{k} + \frac{\kappa}{2} \int_i x^2 ndi, \quad (35)$$

where  $\bar{y} = \int_i y di$  and  $\bar{k} = \int_i k di$ . Finally, lump-sum transfers adjust to balance the government budget constraint:

$$T + (1 - \bar{n})b = 0, \quad (36)$$

where unemployment insurance payments,  $b$ , are given exogenously. This completes the set of model equations. We next define the equilibrium.

A recursive competitive equilibrium is a solution for (i) a set of functions  $\{J(w, \mathbf{s}), V(w, \mathbf{s}), H(w, \mathbf{s}), U(\mathbf{s}), \bar{V}_x(\mathbf{s}), \bar{H}_x(\mathbf{s})\}$ ; (ii) the contract wage  $w^*(\mathbf{s})$ ; (iii) the worker and firm discount factors,  $\epsilon(\mathbf{s})$  and  $\mu(w, \mathbf{s})$ ; (iv) the hiring rate  $x(w, \mathbf{s})$ ; (v) the subsequent period's wage rate  $w(\mathbf{s}')$ ; (vi) the rental rate of capital  $r(\mathbf{s})$ ; (vii) the average wage and hiring rate,  $\bar{w}(\mathbf{s})$  and  $\bar{x}(\mathbf{s})$ ; (viii) the density function of wages across workers  $dG(w, \mathbf{s})$ ; (ix) average consumption and capital,  $\bar{c}(\mathbf{s})$  and  $\bar{k}(\mathbf{s}')$ ; and (x) average employment  $\bar{n}(\mathbf{s})$ . The solution is such that (i)  $w^*(\mathbf{s})$  solves the bargaining condition (22); (ii)  $\epsilon(\mathbf{s})$  and  $\mu(w, \mathbf{s})$  satisfy (23) and (24); (iii)  $x(w, \mathbf{s})$  satisfies the hiring condition (12); (iv)  $w(\mathbf{s}')$  is given by the Calvo process for wages (21); (v)  $r(\mathbf{s})$  satisfies (10); (vi)  $\bar{w}(\mathbf{s})$  and  $\bar{x}(\mathbf{s})$  are defined as in (28) and (15); (vii)  $dG(w, \mathbf{s})$  evolves according to (30); (viii)  $\bar{n}(\mathbf{s})$  evolves according to (33); (ix)  $\bar{c}(\mathbf{s})$  and  $\bar{k}(\mathbf{s}')$  satisfy (34) and (35); and (x) the rental market for capital clears, that is,  $k(\mathbf{s}) = \bar{k}(\mathbf{s})/\bar{n}(\mathbf{s})$ .

### III. Model Characteristics

To gain some intuition, we first present some of the key steady-state conditions and then describe several of the key relationships that help characterize the dynamics outside the steady state.<sup>13</sup>

#### A. Steady State

Here we describe the steady state for the labor market, conditional on the steady-state marginal product of labor  $\tilde{a}$ , where the tilde denotes the steady-state value of a variable. Note that  $\tilde{a}$  is determined as in the

<sup>13</sup> The extended appendix in the working paper version presents the complete steady-state model, as well as the detailed derivation of the key log-linear relationships.

conventional neoclassical growth model and is independent of the labor market equilibrium.

The key labor market variables are the hiring rate  $\tilde{x}$ , the wage  $\tilde{w}$ , the job-finding probability  $\tilde{p}$ , unemployment  $\tilde{u}$ , and vacancies  $\tilde{v}$ . The key behavioral relations are the hiring condition and the solution for the wage bargain:

$$\kappa\tilde{x} = \beta\left(\tilde{a} - \tilde{w} + \frac{\kappa}{2}\tilde{x}^2 + \rho\kappa\tilde{x}\right) \quad (37)$$

and

$$\tilde{w} = \tilde{\chi}\left(\tilde{a} + \frac{\kappa}{2}\tilde{x}^2 + \kappa\tilde{p}\tilde{x}\right) + (1 - \tilde{\chi})b, \quad (38)$$

with

$$\tilde{\chi} = \frac{\eta}{\eta + (1 - \eta)\tilde{\mu}/\tilde{\epsilon}}, \quad \tilde{\mu} = \frac{1}{1 - \lambda\beta}, \quad \tilde{\epsilon} = \frac{1}{1 - \rho\lambda\beta},$$

where  $\tilde{\chi}$  is the steady-state effective worker bargaining power (after we take into account horizon effects on bargaining),  $\tilde{\mu}$  is the steady-state firm discount factor, and  $\tilde{\epsilon}$  is the steady-state worker discount factor. Within a local region of the steady state, the hiring condition relates  $\tilde{x}$  inversely to  $\tilde{w}$ , since the net marginal benefit to the firm of expanding employment is decreasing in both of these variables. Conversely, the wage equation relates  $\tilde{w}$  positively to  $\tilde{x}$ , since firm surplus is increasing in this variable whereas the worker's surplus from the match is decreasing, everything else equal. In addition,  $\tilde{w}$  varies positively with  $\tilde{p}$  since the worker's outside option is increasing in the job-finding probability. Note that the staggered contracting structure affects the steady state only via the impact on the effective bargaining power parameter  $\tilde{\chi}$ . As we discussed earlier, the different horizon of the firm and worker has the effect of reducing the worker's relative bargaining power, which, everything else equal, leads to a lower wage and higher hiring rate.

Next we observe that in the steady state the hiring rate equals the job separation rate:

$$\tilde{x} = 1 - \rho. \quad (39)$$

Equations (37), (38), and (39) thus determine  $\tilde{x}$ ,  $\tilde{w}$ , and  $\tilde{p}$ . Given  $\tilde{x}$  and  $\tilde{p}$ ,  $\tilde{u}$  is pinned down by the condition that new hires by firms must equal the number of unemployed workers who find jobs:

$$\tilde{x}(1 - \tilde{u}) = \tilde{p}\tilde{u}. \quad (40)$$

Finally, the condition that the new matches are given by the matching function pins down vacancies as follows:

$$\tilde{p}\tilde{u} = \sigma_m \tilde{u}^\sigma \tilde{v}^{1-\sigma}. \quad (41)$$

### B. Wage/Hiring Dynamics and Spillover Effects

To gain some intuition, we next derive log-linear versions of the two central equations of the model that govern labor market dynamics outside the steady state: the relations for wages and hiring. Let  $\hat{z}$  denote the percentage deviation of variable  $z$  from its steady-state value  $\tilde{z}$ . Then, log-linearizing the first-order condition for Nash bargaining, equation (25), yields a first-order forward-looking equation for the contract wage,

$$\hat{w}^* = (1 - \tau) \hat{w}'(w^*, \mathbf{s}) + \tau E \hat{w}^{*'}, \quad (42)$$

with

$$\tau = \frac{(\rho\beta\lambda)\psi}{1 + (\rho\beta\lambda)\psi},$$

where  $\psi = \tilde{\chi}\tilde{\mu} + (1 - \tilde{\chi})\tilde{\epsilon}$ , and the forcing variable  $\hat{w}'(w^*, \mathbf{s})$  is given by

$$\begin{aligned} \hat{w}'(w^*, \mathbf{s}) = & \tilde{\chi}[\varphi_a \hat{a} + \varphi_x \hat{x}(w^*, \mathbf{s})] + (1 - \tilde{\chi})\varphi_p E[\hat{p} + \hat{H}_x(\mathbf{s}') \\ & + \hat{\Lambda}(\mathbf{s}, \mathbf{s}')] + \hat{\Phi}(w^*, w^{*'}, \mathbf{s}, \mathbf{s}'), \end{aligned} \quad (43)$$

with

$$\hat{\Phi}(w^*, w^{*'}, \mathbf{s}, \mathbf{s}') = \varphi_\chi E[\hat{\chi}(w^*, \mathbf{s}) - \rho\beta\hat{\chi}(w^{*'}, \mathbf{s}')], \quad (44)$$

where the coefficients  $\varphi_a$ ,  $\varphi_x$ ,  $\varphi_p$ , and  $\varphi_\chi$  depend on the primitive model parameters and steady-state values, as shown in Appendix B.

As in the conventional literature on time-dependent wage and price contracting (Taylor 1980; Calvo 1983), the contract wage depends on an expected discounted sum of the target under perfectly flexible adjustment, in this case  $\hat{w}'(w^*, \mathbf{s})$ , as iterating forward equation (42) suggests. The target  $\hat{w}'(w^*, \mathbf{s})$  is the first-order approximation of the period-by-period Nash bargaining solution for wages, adjusted for the horizon effect. The first term in brackets on the right side of equation (43) is the log-linear approximation of the worker's contribution to the match and the second is the log-linear approximation of the worker's opportunity cost. The horizon effect enters in two ways: First, the effective bargaining weight  $\tilde{\chi}$  replaces the primitive bargaining weight  $\eta$  in determining the relative input on the target wage of the worker's contribution versus the worker's opportunity cost. Second, fluctuations in the effective bargaining weight can influence  $\hat{w}'(w^*, \mathbf{s})$ , as captured by the

term  $\hat{\Phi}(w^*, w^{*'}, \mathbf{s}, \mathbf{s}')$ . As we show shortly, though, under our calibration, the impact of the horizon effect on the path of the target wage is small:  $\tilde{\chi}$  is close to  $\eta$  and  $\hat{\Phi}(w^*, w^{*'}, \mathbf{s}, \mathbf{s}')$  does not fluctuate much.

Note that the target wage  $\hat{w}^o(w^*, \mathbf{s})$  is computed taking as given that all other firms and workers in the economy are operating on multiperiod wage contracts. When we move from partial equilibrium to general equilibrium, it is convenient to distinguish between  $\hat{w}^o(w^*, \mathbf{s})$  and  $\hat{w}^o$ , the target wage that would arise if *all* firms and workers were negotiating a period-by-period wage contract (again, adjusting for the horizon effect). By log-linearizing equation (27), we obtain the following relation between  $\hat{w}^o(w^*, \mathbf{s})$  and  $\hat{w}^o$ :

$$\hat{w}^o(w^*, \mathbf{s}) = \hat{w}^o + \frac{\tau_1}{1 - \tau} E(\hat{w}' - \hat{w}^{*'}) + \frac{\tau_2}{1 - \tau} (\hat{w} - \hat{w}^*), \quad (45)$$

with

$$\hat{w}^o = \tilde{\chi}\varphi_a\hat{a} + [\tilde{\chi}\varphi_x + (1 - \tilde{\chi})\varphi_p]\hat{x} + (1 - \tilde{\chi})\varphi_p\hat{p} + \hat{\Phi} \quad (46)$$

and

$$\hat{\Phi} = \varphi_\lambda E[\hat{\chi}(\hat{w}, \mathbf{s}) - (\rho - \tilde{p})\beta\hat{\chi}(\hat{w}', \mathbf{s}')], \quad (47)$$

where  $\tau_1$  and  $\tau_2$  depend on model primitives, as Appendix B shows.

The difference between  $\hat{w}^o(w^*, \mathbf{s})$  and  $\hat{w}^o$  reflects the influence of spillovers of economy-wide average wages on the individual firm and worker bargain. These spillovers work through their effect on the target wage  $\hat{w}^o(w^*, \mathbf{s})$  and are measured by the gap between the average wage  $\hat{w}$  and the contract wage  $\hat{w}^*$ . As we show in the next section, these spillovers magnify the wage stickiness that comes from a multiperiod contract. The spillover that is most important quantitatively reflects the impact of market wages on the worker's outside option and is captured by the second term in equation (45). If, everything else equal,  $E\hat{w}'$  exceeds  $E\hat{w}^{*'}$ , opportunities are unusually good for workers expecting to move into employment next period. This raises the target wage  $\hat{w}^o(w^*, \mathbf{s})$  and in turn the contract wage  $\hat{w}^*$ . The reverse happens if  $E\hat{w}'$  is below  $E\hat{w}^{*'}$ . In this way stickiness in market wages introduces inertia into the individual wage bargain. The other types of spillovers, captured by the third term, work through the impact of the wage differential on the firm's relative hiring rate and are not important quantitatively.

We can now derive a set of simple difference equations for wages and hiring. The log-linearized wage index is given by

$$\hat{w} = (1 - \lambda)\hat{w}^* + \lambda\hat{w}_{-1}. \quad (48)$$

Combining this expression with the equations that define the evolution

of the contract wage then yields the following second-order difference equation for the aggregate wage:

$$\hat{w} = \gamma_b \hat{w}_{-1} + \gamma_o \hat{w}^o + \gamma_f E\hat{w}', \quad (49)$$

where

$$\begin{aligned} \gamma_b &= (1 + \tau_2)\phi^{-1}, \\ \gamma_o &= \varsigma\phi^{-1}, \\ \gamma_f &= (\tau\lambda^{-1} - \tau_1)\phi^{-1}, \\ \phi &= (1 + \tau_2) + \varsigma + (\tau\lambda^{-1} - \tau_1), \\ \varsigma &= (1 - \lambda)(1 - \tau)\lambda^{-1}, \end{aligned}$$

with  $\gamma_b + \gamma_o + \gamma_f = 1$ . Note that the forcing variable in the difference equation is the spillover-free target wage  $\hat{w}^o$ , that is, the target wage if all firms are bargaining period by period, adjusted for the horizon effect (see eq. [46]).

Owing to staggered contracting,  $\hat{w}$  depends on the lagged wage  $\hat{w}_{-1}$  as well as the expected future wage  $E\hat{w}'$ . Solving out for the reduced form of equation (49) yields an expression that relates the wage to the lagged wage and a discounted stream of expected future values of  $\hat{w}^o$ . Note that the spillover effects, measured by  $\tau_1$  and  $\tau_2$ , work to reduce the relative importance of the expected future wage relative to the lagged wage (by reducing  $\gamma_f$  relative to  $\gamma_b$ ). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, e.g., Woodford 2003).

Note that the formulation nests the conventional period-by-period wage bargaining setup. As  $\lambda$  converges to zero (the case of period-by-period wage bargaining), both  $\gamma_b$  and  $\gamma_f$  go to zero and  $\gamma_o$  goes to unity, implying that  $\hat{w}$  simply tracks  $\hat{w}^o$  in this instance. Further, as we showed in Section II.D, the horizon effect on bargaining disappears in that case, meaning that  $\hat{w}^o$  becomes identical to the wage that would arise economy-wide under period-by-period bargaining.

Finally, log-linearizing the difference equation for the hiring rate (12) and aggregating economy-wide yields

$$\hat{x} = E[\boldsymbol{\kappa}_a \hat{a}' - \boldsymbol{\kappa}_w \hat{w}' + \hat{\Lambda}(\mathbf{s}, \mathbf{s}') + \beta \hat{x}'], \quad (50)$$

where the expressions for the coefficients  $\boldsymbol{\kappa}_a$  and  $\boldsymbol{\kappa}_w$  are reported in Appendix B. The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else

equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

A complete presentation of the log-linear equations of the model appears in Appendix B.

#### IV. Model Evaluation

##### A. Calibration

We choose a monthly calibration in order to properly capture the high rate of job finding in U.S. data. There are 12 parameters to which we need to assign values. Five are conventional in the business cycle literature: the discount factor,  $\beta$ , the depreciation rate,  $\delta$ , the “share” parameter on capital in the Cobb-Douglas production function,  $\alpha$ , the autoregressive parameter of the technology shock,  $\rho_z$ , and the standard deviation of the technology shock,  $\sigma_z$ . We use conventional values for all these parameters:  $\beta = 0.99^{1/3}$ ,  $\delta = 0.025/3$ ,  $\alpha = 0.33$ ,  $\rho_z = 0.95^{1/3}$ , and  $\sigma_z = 0.0075$ .<sup>14</sup> Note that, in contrast to the frictionless labor market model, the term  $1 - \alpha$  does not necessarily correspond to the labor share, since the labor share will in general depend on the outcome of the bargaining process. However, because a wide range of values of the bargaining power imply a labor share just below  $1 - \alpha$ , here we simply follow convention by setting  $1 - \alpha = 2/3$ .<sup>15</sup>

There are an additional six parameters that are specific to the conventional search and matching framework: the job survival rate,  $\rho$ , the elasticity of matches to unemployment,  $\sigma$ , the matching function constant,  $\sigma_m$ , the bargaining power parameter,  $\eta$ , the adjustment cost parameter,  $\kappa$ , and the unemployment flow value,  $b$ .

We choose the average monthly separation rate  $1 - \rho$  on the basis of the observation that jobs last about  $2\frac{1}{2}$  years. Therefore, we set  $\rho = 1 - 0.035$ . We choose the elasticity of matches to unemployment,  $\sigma$ , to be equal to 0.5, the midpoint of values typically used in the literature.<sup>16</sup> This choice is within the range of plausible values of 0.5–0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then note that the parameter  $\sigma_m$  can be normalized. Larger values of this parameter result only in smaller values of average vacancies without affecting the steady-state properties or the dynamics of the model (see eq. [41] in the description

<sup>14</sup> The parameter  $\sigma_z$  is chosen to target the standard deviation of output.

<sup>15</sup> In our calculations,  $1 - \alpha = 0.667$  and the labor share equals 0.646.

<sup>16</sup> The values for  $\sigma$  used in the literature are 0.24 in Hall (2005*b*); 0.4 in Blanchard and Diamond (1989), Merz (1995), and Andolfatto (1996); 0.45 in Mortensen and Nagypal (2007); 0.5 in Farmer (2006) and Hagedorn and Manovskii (2008); and 0.72 in Shimer (2005).

of the steady state). We normalize  $\sigma_m$  to one. To maintain comparability with much of the existing literature, we set the bargaining power parameter  $\eta$  to be equal to 0.5.<sup>17</sup> One of the few studies that provide direct estimates is Flinn (2006), which finds a point estimate of 0.4, close to the value we use. We then use the adjustment cost parameter,  $\kappa$ , and the flow unemployment value,  $b$ , to target the average job-finding probability,  $\tilde{p}$ , and the value of  $\bar{b}$ , defined as the ratio of the unemployment flow value,  $b$ , to the steady-state flow contribution of the worker to the match,  $\tilde{a} + (\kappa/2)\tilde{x}^2$ . We choose  $\tilde{p} = 0.45$  to match recent estimates of the U.S. average monthly job-finding rate (Shimer 2005). Perhaps most controversial is the choice of  $\bar{b}$ . We follow much of the literature by assuming that the value of nonwork activities is far below what workers produce on the job (see Hall [2005*c*] for a brief discussion). In particular, we specifically follow Shimer and Hall and choose  $\bar{b} = 0.4$ . Since we have normalized steady-state technology  $\tilde{z}$  to one, this requires setting  $b = 1.46$  and  $\kappa = 148.2$ . This parameterization implies a ratio of adjustment costs to output equal to 1 percent. In addition, under the interpretation of  $b$  as unemployment benefits, it implies a steady-state replacement ratio of 0.42 (since the steady-state ratio of the wage to the worker's contribution to the job is 0.945).

Finally, there is one parameter that is specific to this model: the probability  $\lambda$  that a firm may not renegotiate the wage. We pick  $\lambda$  to match the average frequency of wage contract negotiations. While there is no systematic direct evidence on the frequency of wage negotiations, Taylor (1999) argues that in most medium- to large-sized firms, wages are typically adjusted once per year. He also argues that this pattern characterizes union workers as well as nonunion workers, including in the latter workers who do not have formal employment contracts. In addition, on the basis of microeconomic data on hourly wages, Gottschalk (2005) concludes that wage adjustments are most common a year after the last change. This evidence of course applies primarily to base pay. There are, however, other components such as bonuses that might be adjusted more frequently over the year, though it is very unclear how important these adjustments might be in practice. Nonetheless, to be conservative, for our baseline case we set  $\lambda = 8/9$ , implying that wage contracts are renegotiated on average once every 3 quarters. We then

<sup>17</sup> Flinn (2006) estimates the bargaining power parameter to be in this range. We find that reasonable perturbations of this value (e.g., from 0.4 to 0.6) have no perceptible effects on our quantitative results. In contrast to much of the literature, we cannot appeal to the Hosios (1990) condition to constrain the parameter in this region since this condition does not generate efficiency within our framework. Because we have costs of adjusting employment as opposed to vacancies, efficiency requires that workers' wages are driven to workers' opportunity cost.

TABLE 1  
VALUES OF PARAMETERS

Discount factor	$\beta$	.997
Capital depreciation rate	$\delta$	.008
Production function parameter	$\alpha$	.33
Technology autoregressive parameter	$\rho_z$	.983
Technology standard deviation	$\sigma_z$	.0075
Survival rate	$\rho$	.965
Elasticity of matches to unemployment	$\sigma$	.5
Bargaining power parameter	$\eta$	.5
Matching function constant	$\sigma_m$	1
Adjustment cost parameter	$\kappa$	148.2
Unemployment flow value	$b$	1.46
Renegotiation frequency	$\lambda$	.889

consider the case of a 4-quarter average contract length as a robustness exercise.

Note finally that as we discussed in the previous section, the implied steady-state horizon-adjusted bargaining parameter  $\tilde{\chi}$  does not vary much from the primitive parameter  $\eta$  (0.44 vs. 0.50). Our parameterization is summarized in table 1.

### B. Results

We judge the model against quarterly U.S. data from 1964:1 to 2005:1. For series that are available monthly, we take quarterly averages. Since the artificial series that the model generates are based on a monthly calibration, we also take quarterly averages of these data.

Most of the data are from the Bureau of Labor Statistics. All variables are measured in logs. Output  $y$  is production in the nonfarm business sector. The labor share  $ls$  and output per worker  $y/n$  are similarly from the nonfarm business sector. The wage  $w$  is average hourly earnings of production workers in the private sector, deflated by the consumer price index. Employment  $n$  is all employees in the nonfarm business sector. Unemployment  $u$  is civilian unemployment 16 years old and over. Vacancies  $v$  are based on the help-wanted advertising index from the Conference Board. Consumption  $c$  is real personal consumption expenditures of nondurables. Investment  $i$  is equal to the sum of real private investment plus real personal consumption of durables. Finally, the data are Hodrick-Prescott filtered with a conventional smoothing weight.

We examine the behavior of the model taking the technology shock as the exogenous driving force. To illustrate how the wage contracting process affects model dynamics, we first examine the impulse responses of the model economy to a unit increase in total factor productivity. The solid line in each panel of figure 1 illustrates the response of the respective variable for our model. For comparison, the dotted line re-

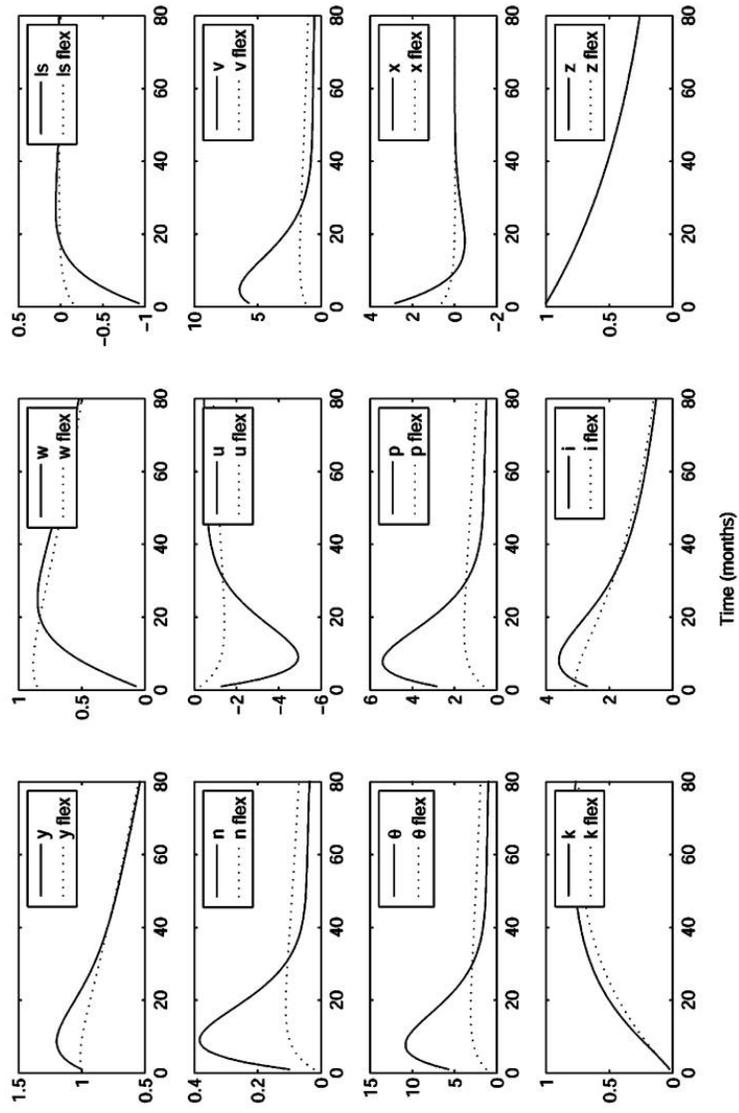


FIG. 1.—Impulse responses to a technology shock

TABLE 2  
AGGREGATE STATISTICS

	<i>y</i>	<i>w</i>	<i>ls</i>	<i>n</i>	<i>u</i>	<i>v</i>	$\theta$	<i>a</i>	<i>i</i>	<i>c</i>
A. U.S. Economy, 1964:1–2005:1										
Relative standard deviation	1.00	.52	.51	.60	5.15	6.30	11.28	.61	2.71	.41
Autocorrelation	.87	.91	.73	.94	.91	.91	.91	.79	.85	.87
Correlation with <i>y</i>	1.00	.56	-.20	.78	-.86	.91	.90	.71	.94	.81
B. Model Economy, $\lambda = 0$ (Flexible Wages)										
Relative standard deviation	1.00	.87	.09	.10	1.24	1.58	2.72	.93	3.11	.37
Autocorrelation	.81	.81	.58	.92	.92	.86	.90	.78	.80	.85
Correlation with <i>y</i>	1.00	1.00	-.54	.59	-.59	.98	.92	1.00	.99	.93
C. Model Economy, $\lambda = 8/9$ (3 Quarters)										
Relative standard deviation	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Autocorrelation	.84	.95	.65	.90	.90	.82	.88	.76	.86	.86
Correlation with <i>y</i>	1.00	.66	-.56	.77	-.77	.91	.94	.97	.99	.90
D. Model Economy, $\lambda = 11/12$ (4 Quarters)										
Relative standard deviation	1.00	.48	.58	.44	5.68	7.28	12.52	.64	3.18	.34
Autocorrelation	.85	.96	.68	.91	.91	.86	.90	.74	.88	.86
Correlation with <i>y</i>	1.00	.55	-.59	.78	-.78	.93	.95	.95	.99	.90

ports the response of the conventional flexible wage model with period-by-period Nash bargaining (obtained by setting  $\lambda = 0$ ).

Observe that in the conventional case with period-by-period wage adjustment, the response of employment is relatively modest, confirming the arguments of Hall and Shimer. There is also only a modest response of other indicators of labor market activity, such as vacancies,  $v$ , unemployment,  $u$ , labor market tightness,  $\theta = v/u$ , and the hiring rate,  $x$ . Wages, by contrast, adjust quickly. The resulting small adjustment of employment leads to output dynamics that closely mimic the technology shock.

By contrast, in the model with staggered multiperiod contracting, there is a smooth drawn-out adjustment in wages, directly a product of the staggered multiperiod contracting. As a consequence, the hiring rate jumps sharply in the wake of the technology shock along with the measures of labor market activity. A substantial rise in employment follows, certainly as compared to the conventional flexible wage case. The lagged rise in employment leads to a hump-shaped response of output; that is, output continues to rise for several periods before reverting to trend, in contrast to the technology shock, which reverts immediately. The quadratic adjustment costs of hiring, along with the wage rigidity, contribute to the persistence of the output response.

We next explore how well the model economy is able to account for the overall volatility in the data. Panel A in table 2 reports the standard deviation, autocorrelation, and contemporaneous correlation with out-

put for the 10 key variables in the U.S. economy. The standard deviations are normalized relative to output.

To provide a benchmark, panel B presents statistics from the model economy for the standard case of period-by-period wage bargaining. As in Shimer (2005), the model does not come close to generating the relative volatility of the labor market variables that appears in the data.

In contrast, panel C shows that the model economy for the baseline case (3 quarters) appears to capture well most of the basic features of the data. It comes reasonably close to matching the relative volatilities and comovements of the key indicators of labor market activity, including unemployment  $u$ , vacancies  $v$ , and the tightness measure  $\theta$ . These were the variables emphasized in the Hall/Shimer analysis. The model captures only about 60 percent of the relative volatility of employment. However, here it is important to keep in mind that the framework abstracts from labor force participation, a nontrivial source of cyclical employment volatility. Some further evidence of its overall plausibility is that the model comes close to accounting for the relative behavior of both consumption and investment.

A distinguishing feature of our analysis is that we appear to capture wage dynamics. Note that we come very close to matching the relative volatility of wages (0.56 vs. 0.52 in the data), their autocorrelation (0.95 vs. 0.91 in the data), and the contemporaneous correlation of wages with output (0.66 vs. 0.56 in the data).

As we noted earlier, we assumed 3-quarter average length wage contracts for our baseline case to err on the side of caution, even though the evidence suggests that the modal period of wage adjustments is 1 year. In panel D of table 2 we also report statistics based on 4-quarter average length wage contracts. Interestingly, the performance of the model improves overall. Not surprisingly, the enhanced wage rigidity raises the volatilities of the labor market variables. In the end, the model tracks the relative volatilities and comovements of the key labor market variables,  $u$ ,  $v$ , and  $\theta$ , as well as in the baseline case. The model, however, is also now able to capture nearly 3 quarters of the relative volatility of employment.

In table 3 we next consider several variations of the model designed to illustrate the importance of the bargaining spillovers and of the horizon effect on wage bargaining. As we discussed earlier, the inertia in wage dynamics is a product not simply of staggered multiperiod contracting, but also of the spillover effect of economy-wide wages on the individual wage bargain that arises in this kind of environment. To quantify the importance of these spillovers for model dynamics, we simulate the model eliminating the spillover effects on wage dynamics. In particular, we set equal to zero the parameters  $\tau_1$  and  $\tau_2$ , which govern the magnitude of the spillover effects, in equation (49). To be clear,  $\tau_1$

TABLE 3  
THE SPILLOVER EFFECT AND THE HORIZON EFFECT: RELATIVE STANDARD DEVIATIONS

	$y$	$w$	$l_s$	$n$	$u$	$v$	$\theta$	$a$	$i$	$c$
Model economy: baseline	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Model economy: no spillover	1.00	.69	.50	.19	2.50	3.38	5.58	.83	3.13	.36
Model economy: no horizon	1.00	.57	.53	.35	4.53	5.97	10.05	.70	3.20	.34

and  $\tau_2$  are both functions of model primitives that we are holding fixed. Thus the exercise is not a formal counterfactual experiment, but rather an informal way to judge how the spillover mechanism is shaping model dynamics.

As table 3 makes clear, eliminating the spillovers significantly enhances wage flexibility and reduces employment volatility. When the spillovers are removed, the relative volatility of wages jumps nearly 50 percent, from 0.56 to 0.69. Conversely, the relative volatility of employment is reduced roughly in half, from 0.35 to 0.19. The other measures of labor activity— $u$ ,  $v$ , and  $\theta$ —similarly fall by about a half. Overall, as a comparison of tables 2 and 3 suggests, the spillovers are responsible for about a half of the added rigidity in wages relative to the flexible benchmark model and for about two-thirds of the added volatility in the labor market.

The last row of table 3 presents information on the importance of the horizon effect on the wage bargain. The row presents the model statistics for our baseline case, but with the horizon effect shut off (i.e.,  $\chi$  fixed at  $\eta$ ). The relative volatilities remain close to those arising in the baseline case. Thus, as we conjectured, the horizon effect is not that important quantitatively.

Finally, it is interesting to compare our analysis with that of Hagedorn and Manovskii (2008), who argue that an alternative calibration of the Mortensen-Pissarides model can account for labor market volatility. In particular, these authors find parameters that allow the model to match the low elasticity of wages with respect to productivity present in the data. From micro data, they estimate this elasticity to be 0.47. To have the model match it, they require a very low value of  $\eta$ , the bargaining power of workers, and a very high value of  $\bar{b}$ , the relative steady-state flow value of unemployment, as compared to what is conventional in the literature. In particular, they require  $\eta$  very close to zero, well below the conventionally used value of 0.5, as well as Flinn's (2006) estimate of 0.4. In addition, they require  $\bar{b}$  close to unity, well above Shimer's (2005) and Hall's (2005*c*) preferred value of 0.4. A value of  $\bar{b}$  close to unity, of course, implies that workers are nearly indifferent between

TABLE 4  
WAGES AND LABOR SHARE STATISTICS

	$el(w, a)$	$\text{corr}(w, a)$	$\sigma_w/\sigma_a$
U.S. data	.53	.62	.85
Mortensen-Pissarides baseline	.98	1.00	.98
Hagedorn-Manovskii	.49	1.00	.49
Gertler-Trigari	.50	.63	.79
	$el(ls, a)$	$\text{corr}(ls, a)$	$\sigma_b/\sigma_a$
U.S. data	-.50	-.60	.83
Mortensen-Pissarides baseline	-.02	-1.00	.02
Hagedorn-Manovskii	-.51	-1.00	.51
Gertler-Trigari	-.50	-.63	.80

employment and unemployment. The overall calibration effectively makes labor supply highly elastic, enabling the model to have large employment movements with moderate wage adjustments. Nonetheless, the resulting calibration is not without controversy.<sup>18</sup>

Interestingly, we find from our macro data that the wage elasticity with respect to labor productivity is 0.53, which is very close to the estimate that Hagedorn and Manovskii obtained from micro evidence. However, as we suggested earlier, we stick with conventional values of  $\eta$  and  $\bar{b}$  and instead introduce wage sluggishness by appealing to staggered multiperiod contracts. Further, as opposed to picking parameters to match the wage elasticity, we calibrate the average duration of wage contracts to match the evidence. We then ask how well the model explains the wage elasticity (along with other volatilities). It turns out that the model does very well on this accounting, generating a wage elasticity of 0.50, nearly identical to what our data suggest.

In addition, as we observed in table 2, our model does well at explaining the overall cyclical volatility of wages, including the comovement with aggregate activity as well as the relative volatility. However, the Hagedorn-Manovskii model does not do well on this dimension, even though it is calibrated to match the wage elasticity with respect to productivity. How can this be? Note that since this elasticity,  $el(w, a)$ , is effectively a regression coefficient from the regression of log wages on log productivity, it equals the product of the correlation  $\text{corr}(w, a)$  and the relative standard deviations  $\sigma_w/\sigma_a$ . Since their calibration fixes only the product of these two moments, it need not do well at matching them individually. This turns out to be the case, as we show next.

Table 4 compares values of  $el(w, a)$ ,  $\text{corr}(w, a)$ , and  $\sigma_w/\sigma_a$  against U.S. data for three models: the conventional Mortensen-Pissarides model,

<sup>18</sup> In addition to having values of  $\eta$  and  $\bar{b}$  that are at variance with the literature, Hornstein, Krusell, and Violante (2005) note that Hagedorn and Manovskii's calibration implies suspiciously large employment effects from changes in unemployment insurance.

the framework based on the Hagedorn-Manovskii calibration, and our baseline model with staggered wage contracting (Gertler-Trigari). While the Hagedorn-Manovskii model captures  $el(w, a)$  by construction, it misses badly on the other two moments. The correlation between wages and productivity is too high (unity vs. 0.62 in the data) whereas the relative volatility of wages is too low (0.49 vs. 0.85). The former outcome is due to the period-by-period Nash bargaining that ties aggregate wage movements to current-period productivity. The latter result arises from the low bargaining power of workers, which forces their wages close to their reservation values. Thus while the Hagedorn-Manovskii model by construction matches  $el(w, a)$ , it does so by inducing offsetting errors in  $\text{corr}(w, a)$  and  $\sigma_w/\sigma_a$ .

By contrast, our model does well at matching not only the wage elasticity but also the correlation of wages and productivity, as well as the relative volatility. The staggered contract structure works to dampen the correlation between productivity and wages. At the same time, because workers have more bargaining power than in the Hagedorn-Manovskii calibration, wages are more sensitive to productivity movements, permitting the model to match the data. Again, we stress that our model is calibrated to match the average duration of contract lengths. It is therefore not by construction that we match the wage elasticity, in contrast to Hagedorn-Manovskii.

Finally, the bottom panel of table 4 shows that similar conclusions apply for the volatility of the labor share. While the Hagedorn-Manovskii model does not explain all the relevant moments well, our framework does.

## V. Issues

We next address several issues involving the robustness of the assumptions that underlie our analysis.

### A. *Bargaining Set*

A key maintained hypothesis in our analysis is that workers and firms can expect that they will not want to voluntarily dissipate their relationship over the life of their relationship. This assumption simplifies how both parties form expectations when they enter relationships. Here we demonstrate that this condition holds to a reasonable approximation. Put differently, under our parameterization, wages have a negligible probability of falling outside the bargaining set. Intuitively, given our Poisson process for contract adjustment, only a very small fraction of contracts will have a duration sufficiently long for the wage to move out of the bargaining set.

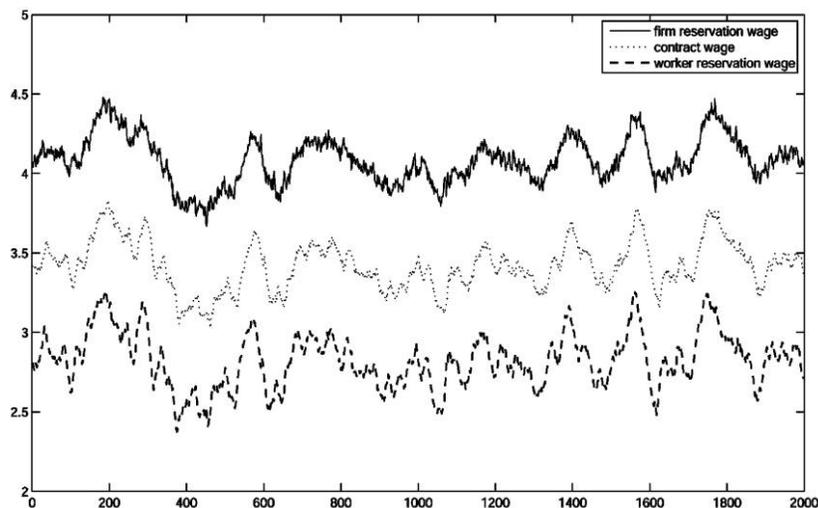


FIG. 2.—Simulated bargaining set versus contract wage of 9 months' ex post duration

The lower and upper limits of the bargaining set are given by, respectively, the reservation wage of the worker and the reservation wage of the firm. We compute these values by finding the wage for the current contract that makes the expected surplus zero for each party, given that the wage will be renegotiated once the contract expires. If the wage stays in the bargaining set as we have defined it, then both parties are willing to stay in the relationship until the next round of contract negotiations. If it does not, then one party is not willing.

To analyze how likely the wage is to stay in the bargaining set, we first generate artificial time series from the model and then compute a time series for the firm and worker reservation wages and the contract wage. We then check whether a contract wage set  $\tau$  periods earlier lies within the current bargaining set.

In figure 2, for example, we consider a contract that has been in place for the average time of 9 months. As the figure shows, at any time  $t$  for the simulated series, the contract wage from 3 quarters earlier lies safely in the bargaining set.

In figure 3, we consider wage contracts that have been in place for 40 months. Even in this case the contract wage stays in the bargaining set, though it edges closer to the boundaries than in the 9-month case. However, under our baseline calibration, the probability a contract will remain intact for 40 months is less than 1 percent:  $\lambda^{40} = (1 - 1/9)^{40} = 0.0089$ . Thus, at the time of a new contract negotiation, there

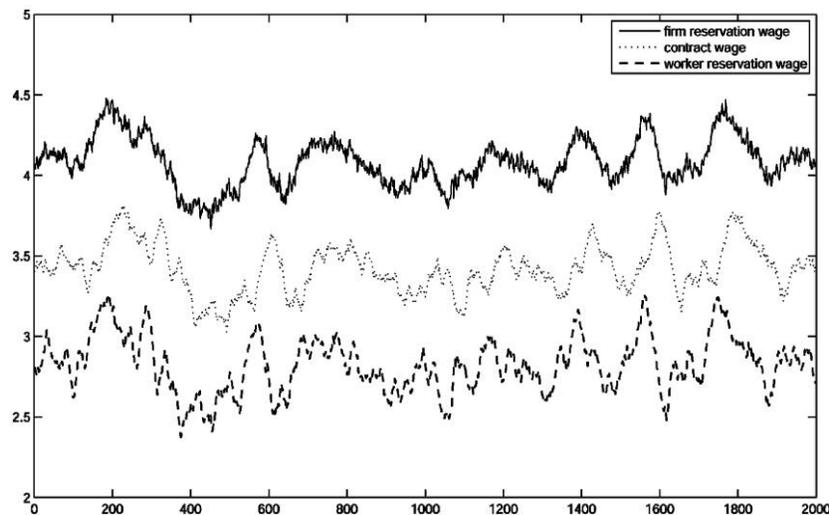


FIG. 3.—Simulated bargaining set versus contract wage of 40 months' ex post duration

is less than a 1 percent chance that the contract wage will eventually fall out of the bargaining set.<sup>19</sup>

Note that we could have instead employed a “truncated” Poisson process in which the probability of renegotiation is Poisson for  $T$  periods and then is unity for any contract that has lasted  $T$  periods. Assuming that  $T$  is not too large—say 40 months or less—then we could guarantee that firms never go out of the bargaining set with certainty.<sup>20</sup> Given our calibration, however, the truncated Poisson generates results nearly identical to what our simple Poisson process gives. The gain from using the simple process is that the algebra stays simple.

#### B. *Welfare Costs of Multiperiod Contracts*

We have been appealing partly to realism and partly to the notion that bargaining is costly to justify firms and workers negotiating wage contracts only infrequently. It is reasonable to ask, however, how much the

<sup>19</sup> We generate an artificial time series of 120,000 observations, even though to preserve the clarity of figs. 2 and 3 we plot only 2,000 of them. It is possible in theory that very large shocks may take the wage out of the bargaining set. However, the probability of these extreme realizations is small enough that they do not occur over the length of our simulation.

<sup>20</sup> We could also allow for idiosyncratic shocks using this approach. In this instance  $T$  would have to be less than 40 months, though it could be significantly longer than 9 months, as inspection of figs. 2 and 3 suggests.

two parties would gain if they could costlessly negotiate wages on a period-by-period basis. The answer is very little.

We consider the following experiment. We first compute the firm surplus and the worker surplus for a typical firm/worker pair operating on multiperiod contracts and currently renegotiating. We then ask how the surplus measures differ for a firm/worker pair that deviates from all the other pairs by operating on period-by-period contracts.

Combining the recursive formulation for  $J$  in equation (9) with the optimality conditions for capital and hiring, equations (10) and (11), leads to the following simple expression for the match surplus of a firm that is currently renegotiating:

$$\begin{aligned} J(w^*, \mathbf{s}) &= a - w^* - \frac{\kappa}{2} x(w^*, \mathbf{s})^2 + [\rho + x(w^*, \mathbf{s})] \kappa x(w^*, \mathbf{s}) \\ &= a - w^* + \frac{\kappa}{2} x(w^*, \mathbf{s})^2 + \rho \kappa x(w^*, \mathbf{s}). \end{aligned} \quad (51)$$

Consider now the worker's surplus in equation (19). Let us define  $\Theta \equiv b + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}') \rho \bar{H}_x(\mathbf{s}') | \mathbf{s}\}$  as the component of the surplus that is independent of current and expected future wage bargains. We can then express the surplus of a worker who is currently renegotiating as

$$H(w^*, \mathbf{s}) = w^* + \beta E\{\Lambda(\mathbf{s}, \mathbf{s}') \rho H(w', \mathbf{s}') | w^*, \mathbf{s}\} - \Theta. \quad (52)$$

The choice of contract length, of course, influences the volatility of wages and hiring. Since  $J$  is linear in  $w^*$  and quadratic in  $x$ , the firm cares only about the volatility of hiring. In this regard, there are two offsetting effects: First, as the third term in the top line of equation (51) indicates, an increase in the variability of the hiring rate raises adjustment costs, which reduces  $J$ . Second, and conversely, an increase in the variability of  $x$  raises the expected discounted value of future profits, as the last term suggests, implying in turn an increase in  $J$ . When expected discounted labor productivity net of wage increases, for example, the firm raises its hiring rate, making the latter a signal of the former. By doing more hiring, further, the firm expands its expected size, further raising overall expected profits. The combined effect is that expected future profits are increasing and convex in  $x$ .

As equation (51) shows, the positive effect of the variability of  $x$  on expected future profits outweighs the negative effect on current adjustments. Thus, overall,  $J$  is increasing and convex in  $x$ , which suggests that, everything else equal, the firm prefers volatility in the hiring rate. That  $J$  is convex in the hiring rate with convex adjustment costs is a standard result in the literature. In the context of our model, it suggests that firms might actually prefer multiperiod contracts since the smoothed behavior of the wage raises hiring volatility.

TABLE 5  
FIRM AND WORKER SURPLUS VALUES

	STEADY- STATE VALUE (1)	CHANGE FROM STEADY STATE Aggregate: $\lambda = 8/9$		CHANGE FROM STEADY STATE Aggregate: $\lambda = 0$
		Firm: $\lambda = 8/9$ (2)	Firm: $\lambda = 0$ (3)	Firm: $\lambda = 0$ (4)
Firm surplus, $J$	5.20	.15%	.10%	.01%
Worker surplus, $H$	4.09	-2.40%	-2.36%	-2.19%
Nash product, $NP$	4.61	-1.18%	-1.18%	-1.14%

The worker does care about the volatility of the wage. As equation (52) indicates,  $H$  depends on how its value in the subsequent period covaries with the stochastic discount factor,  $\Lambda(\mathbf{s}, \mathbf{s}')$ . In general, this covariance depends on the covariance of the expected future path of wages with the whole term structure of stochastic discount factors. To the extent that wages move procyclically and thus vary inversely with the marginal utility of consumption, wage volatility reduces the worker's surplus from the job. In general, it appears ambiguous whether the worker would prefer multiperiod or period-by-period contracts. While period-by-period contracts increase the sensitivity of the wage to contemporaneous shocks, multiperiod contracts introduce persistent movements in the wage, enhancing the overall unconditional volatility.

To quantify the impact of multiperiod contracting on the firm and the worker, we first take quadratic approximations of (51) and (52) around their respective deterministic steady-state values. We then simulate the model and evaluate the loss under multiperiod contracts versus period-by-period contracts. Column 1 of table 5 reports the deterministic steady-state values of  $J$  and  $H$ , as well as the composite surplus  $NP = H^\eta J^{1-\eta}$ , under the benchmark parameterization. Column 2 reports the percentage change in the unconditional value of each variable induced by macroeconomic volatility for our benchmark case in which all firms are operating on multiperiod contracts. As we have suggested, the firm prefers the volatility, though the gain is very slight: The unconditional value of  $J$  increases by 0.15 percent. For the worker, the change differs in both sign and magnitude: The procyclical wage volatility reduces the worker surplus by 2.40 percent.

We next consider the case in which one firm deviates from the others by negotiating period-by-period contracts. One technical complication emerges because we have used the steady state to pin down the adjustment cost parameter  $\kappa$ . Because the steady state depends on the horizon effect on bargaining (through the effect of multiperiod contracting on the effective bargaining power parameter  $\tilde{\chi}$ ), the implied value of  $\kappa$  for a firm can differ depending on contract length. We accordingly proceed

to do the calculation two different ways: First, for the firm on period-by-period contracts, we keep  $\kappa$  the same as for the other firms by adjusting the primitive bargaining power parameter  $\eta$  to equal  $\tilde{\chi}$ . In this case, the steady state of the firm under period-by-period contracting is identical to that of a firm under multiperiod contracting. Second, we keep  $\eta$  the same across firms and allow  $\kappa$  to vary. Fortunately, because the horizon effect is not quantitatively important under our benchmark calibration, the two approaches yield nearly identical results. Accordingly, table 5 just reports (in col. 3) the results using the first method.

For a firm under period-by-period contracting, the percentage increase in  $J$  stemming from macroeconomic volatility is smaller than in the case of multiperiod contracting. This is not surprising, given that firms prefer hiring volatility, everything else equal, and that this volatility declines under period-by-period contracting. The difference is very small, though: a 0.10 percent gain versus a 0.15 percent gain. For a worker, the reverse is true:  $H$  is larger under period-by-period contracts. Again the margin of difference is small: a 2.36 percent loss relative to the steady state with period-by-period contracting versus a 2.40 percent loss with multiperiod contracting. Since the gain to the worker offsets the loss to the firm, the composite surplus  $NP$  is effectively the same under period-by-period and multiperiod contracting.

Finally, we ask how the parties' bargaining surpluses would be affected if all firms switched to period-by-period contracting. Results are shown in column 4 of table 5. In this case, as we saw earlier, there is an overall decline in aggregate volatility:  $J$  decreases relative to the other cases and  $H$  increases. Overall, the surplus  $NP$  improves. But, again, the magnitudes of the changes are very small: under 0.25 percent in each case.

We conclude that under our benchmark formulation there are no strong incentives for firms and workers acting either alone or in concert with others to switch from multiperiod to period-by-period wage contracting.<sup>21</sup> Key to this result, though, is that as we showed in the previous

<sup>21</sup> For realism and convenience, we have restricted attention to a simple multiperiod contract with a constant wage per period. It has been suggested that a contract in which the wage is declining over time may be preferred to this simple contract. In our view, however, the constraint that new workers are treated the same as (similarly productive) existing workers should apply to the present value of compensation they receive and not simply to the most recent contract wage. Under this interpretation, new hires receive the same expected wage profile as existing workers did when they started the most recent contract. Thus, since changing the timing of wage payments does not change the present value of payments to either existing workers or new hires, there is no benefit to doing so. Note, though, that even under the alternative scenario in which new hires just receive the latest contract wage, the gains from a downward-sloping wage profile are not large: Since the benefits of moving from the simple multiperiod contract to fully flexible period-by-period contracts are small, it is possible to show that so too are the benefits of moving to a more exotic multiperiod contract that is nonetheless still more restrictive than a period-by-period contract.

subsection, the contract wage is almost surely in the bargaining set over the life of the contract. If this were not the case, firms and workers would have a much stronger rationale for wanting to consider period-by-period contracting.

### *C. Evidence*

As we have been noting, a key restriction in our model is that in between contract periods, new hires receive the same wages as existing workers of similar productivity. Here we first discuss existing panel data evidence on the cyclical behavior of new hire wages and then present some additional evidence in support of our hypothesis. We next turn to another kind of evidence on the employment effects of wage contracting that also provides support for our approach.

#### 1. Wages of New Hires

There is a lengthy literature, beginning with Vroman (1977, 1978), that shows that job movers have more cyclical wages than job stayers. Some other notable contributions include Bills (1985) and Barlevy (2001). Vroman's original interpretation of this evidence was that it reflected cyclical movements in the composition of job quality: The fraction of hiring by "high-wage" firms increases during booms and declines during recessions. Other examples of a compositional interpretation of the evidence include Barlevy (2001) and Moscarini and Postel-Vinay (2009).

To the extent that it is compositional effects at work, the new hire wage evidence does not contradict our maintained hypothesis. While workers may transition between high- and low-wage jobs over the cycle, the wages of new hires may still be tied to those of existing workers within the same firm. In this instance wage rigidity within the firm will still amplify the effects of disturbances on hiring.

Pissarides (2007), however, argues that the evidence instead suggests that the wages of new hires within a given firm are more cyclical than those of the existing workers within the same firm. In our view, the evidence is not sufficiently sharp to support this claim for two related reasons: First, in virtually every case the data include evidence only on workers and not on firms; that is, they do not match workers with firms. Thus, it is not possible to directly compare new hires with existing workers in the same firm. Second, in every case, there is not an adequate effort to control for cyclical changes in the composition of job quality. Simple industry controls are not adequate since, as Vroman notes, these compositional effects can arise within industries as well as across.

To illustrate how the compositional effects may underlie the evidence on new hire wage cyclicalities, we draw on some companion empirical

work by Gertler, Huckfeldt, and Trigari (2008). The data in this study pertain to individual workers taken from the Survey of Income and Program Participation (SIPP) over the years 1990–96. It includes four panels lasting roughly 3 years each from 1990, 1991, 1992, and 1993. There is a new sample of individuals in each panel. Each individual within the samples is interviewed every 4 months. In addition, the study restricts attention to males.

An important advantage of the SIPP is that it is a large representative sample. Moreover, the 3-year sampling period for individuals allows for cyclical variation. We focus on the 1990–96 data for several reasons. First, over this period it is possible to obtain good-quality information on both job transitions and job tenure, which is critical to the analysis. Second, the period contains the 1990–91 recession and its aftermath and thus includes considerable fluctuations in aggregate economic activity.

We consider two types of regression equations to evaluate the cyclical sensitivity of new hires' wages: The first follows the literature. The second is a variation that introduces a control for compositional effects. Let  $w_{i,j,t}$  be the real wage of worker  $i$  in firm  $j$  at time  $t$ ,  $X_{i,t}$  be a vector of individual-specific factors (e.g., age, sex, education, experience, tenure, etc.),  $U_t$  the civilian male unemployment rate, and  $I(\text{new}_{i,j,t})$  an indicator variable that takes on a value of unity if the worker is a new hire and zero otherwise. Following the literature, we define a new hire as someone with job tenure under a year. However, the results are robust to using a shorter period of job tenure to define a new hire (e.g., 6 months).

The specification we consider that follows the literature is given by

$$\log(w_{i,j,t}) = \pi'_x \cdot X_{i,t} + \pi_u \cdot U_t + \pi_n \cdot I(\text{new}_{i,j,t}) \cdot U_t + \gamma_i + \epsilon_{i,j,t} \quad (53)$$

where  $\gamma_i$  is an individual fixed effect and  $\epsilon_{i,j,t}$  is an independently and identically distributed error term. A typical finding in the literature is that  $\pi_n < 0$ , implying that new hires have wages that are more procyclical than those of existing workers (since unemployment is countercyclical).<sup>22</sup>

From the above specification, however, it is not possible to tell whether the greater wage cyclicality is due to compositional effects, as originally stressed in the literature, or greater volatility of new hires' wages within a given firm, as argued by Pissarides. While the regression controls for

<sup>22</sup> Beaudry and DiNardo (1991) appeal to implicit contracts theory to propose an alternative specification that relates workers to earlier macroeconomic conditions, in particular, the minimum unemployment rate over their job tenure, as to current unemployment. However, Gertler, Huckfeldt, and Trigari (2008) show that by more carefully controlling for job tenure effects on wages, the minimum unemployment variable loses its explanatory power to current unemployment.

individual heterogeneity via the vector of individual characteristics  $X_{i,t}$ , it does not control for cyclical movements in the composition of job quality for new hires. Suppose, for example, that a highly skilled machinist takes a job as a low-paid cabdriver in a recession and then is reemployed as a high-paid machinist in a boom. In this case there is a cyclical movement in job match quality for the individual that is not captured in equation (53).

A way to modify this specification to control for compositional shifts in match quality is to allow for a separate fixed effect  $\gamma_{ij}$  for each job  $j$ , as follows:

$$\log(w_{i,j,t}) = \pi'_x \cdot X_{i,t} + \pi_u \cdot U_t + \pi_n \cdot I(\text{new}_{i,j,t}) \cdot U_t + \gamma_{ij} + \epsilon_{i,j,t}. \quad (54)$$

The only difference between the two specifications is that in the latter we are allowing the fixed effect to vary as the worker changes jobs. To be clear, the intercept  $\gamma_{ij}$  is a combination of a pure individual effect that is independent of the match and a component that reflects match quality that we cannot separately identify. However, given that the pure individual effect is constant, the difference in  $\gamma_{ij}$  across two jobs will capture differences in match quality.

To separate the fixed effect on wages from changing jobs from the cyclical effect of aggregate labor market conditions on wages, it is necessary that we observe a worker on at least one job spell that lasts more than the contract period. That is, at least one job spell in our data must be long enough to observe the impact of cyclical variation on the wage. If we assume that 1 year is a reasonable benchmark for how long a wage contract lasts, then our data set easily satisfies this criterion.<sup>23</sup>

Table 6 reports the results for the conventional specification in column 1 and for the specification that allows for job-specific fixed effects in column 2. The results for the conventional specification are consistent with the literature. The coefficient interacting new hires with unemployment,  $\pi_n$ , is significantly negative and nearly twice the size in absolute value of the coefficient on unemployment,  $\pi_u$ . The results thus suggest that the percentage response of new hires' wages to a change in unemployment is roughly three times as large as it is for existing workers' wages, which is consistent with findings elsewhere.

Once we allow for firm-specific fixed effects, however, the results change dramatically. The new hire effect on wages is no longer statistically significant. That is, after we control for compositional effects, new hire wages appear no more cyclically sensitive than existing workers'

<sup>23</sup> Of the roughly 18,000 individuals in our sample, 3,989 were "new hires" at some point in the panel. Of these, we observe 3,291 with tenure on at least one job strictly greater than a year.

TABLE 6  
 BASELINE FIXED-EFFECTS SPECIFICATION

	Person Fixed Effect (1)	Person-Job Fixed Effect (2)
Unemployment rate	-.00564*** (.0010)	-.00576*** (.0009)
Unemployment rate · $I(\text{new})$	-.01042*** (.0038)	.00193 (.0037)
Observations	125,941	122,026
Number of groups	17,897	18,234
Minimum number of observations per group	2	2
Maximum number of observations per group	9	9
Mean number of observations per group	7.037	6.692

SOURCE.—SIPP, 1990–93.

\* Significant at 10 percent.

\*\* Significant at 5 percent.

\*\*\* Significant at 1 percent.

wages. In Gertler, Huckfeldt, and Trigari (2008), we show that the results are robust to alternative ways of controlling for composition.

How do we reconcile the results from the two different specifications? If indeed match quality varies across jobs, then the failure to allow for a job-specific fixed effect implies that there is an omitted variable in the first specification that enters the error term. Further, if the net movement from low- to high-quality matches is procyclical, then the error term will be negatively correlated with the variable that interacts new hires with unemployment. Under this interpretation, accordingly, the negative estimate of  $\pi_n$  reflects omitted variable bias due to the failure to control for the composition of job match quality.

We do not claim that these results prove definitely that the relatively high cyclicity of new hire wages is due entirely to compositional effects. But they do suggest that at a minimum the existing literature has failed to adequately account for this possibility. As a result, it is not possible to conclude from this literature in its current form that new hires have more cyclical wages than existing workers within the same firm.

## 2. Employment Effects of Wage Contracting

A different kind of evidence relevant to our analysis assesses whether, by shaping the response of wages, the multiperiod contract structure influences how a firm's hiring responds to aggregate disturbances. The empirical work of which we are aware appears consistent with this prediction. The sharpest results come from a study of the employment and wage dynamics of a panel of Canadian manufacturing firms by Card (1990). The firms Card examined all were on union wage contracts, but the contracts differed by the provisions for indexing to inflation.

The experiment Card considered was to analyze the response of firm employment to the average firm wage, using unexpected inflation as an instrument. Because firms differed in indexing provisions, the surprise inflation generated a dispersion in real wages. Card then found in firm-level regressions that movements in the real wage induced by the unanticipated surprise inflation had a significant effect on employment. In this way, restrictions on wage adjustment induced by the contract structure appear to have influenced the response of hiring to an aggregate shock. These results lend support to our general approach.

Olivei and Tenreiro (2007, 2008) also present evidence that wage contracting affects cyclical employment dynamics using aggregate data. In particular, they show that the response of employment to a monetary policy shock has a strong seasonal component: The effects of shocks early in the year are much weaker than of shocks later on. They then show that seasonal patterns in the adjustment of wage contracts can account for this phenomenon: Wage adjustments from new contracts are more frequent early in the year relative to later on. The frequent wage adjustments early in the year dampen the effects of monetary shocks that occur at this time. While their 2007 paper obtains the results for U.S. data, their follow-up paper shows that the results are robust to data from other industrialized economies.

#### *D. Multiple Shocks*

We used as our baseline macroeconomic framework a real business cycle model with a technology shock as the single driving force, partly to follow the literature and partly to keep the analysis sufficiently simple to focus on the details of the wage contracting structure. This raises the issue (applicable to the entire literature) of whether it makes sense to evaluate the model against unconditional data moments if there are other important shocks besides productivity that drive aggregate economic activity.<sup>24</sup>

It is important to recognize, however, that we are not trying to evaluate how well the model with productivity shocks alone matches the data. Rather, we are instead using this simple single-shock model as a laboratory to evaluate the implications of different wage contracting structures for labor market volatility. Thus, the moment matching exercise in Section IV should be viewed as a check of whether indeed the paradigm is a reasonable laboratory for this purpose. That our baseline model with wage rigidity captures the aggregate statistics reasonably well

<sup>24</sup> If there was an agreed-on way to identify pure productivity shocks, then one could simply look at the data conditional on these shocks. Unfortunately this is not the case, given the difficulty in cleaning off unmeasured factor utilization from the measure of total factor productivity.

suggests that it is a credible framework to use to study how much the performance deteriorates with wage flexibility.

To be more specific, we believe that our simple quantitative approach provides insights that carry over to a more general setting, at least in a “ballpark” sense. Note that we can rewrite our key log-linear relation between the hiring rate and wages, equation (50), as follows:

$$\hat{x} = E[\kappa_a(\hat{y}' - \hat{n}') - \kappa_w\hat{w}' + \hat{\Lambda}(\mathbf{s}, \mathbf{s}') + \beta\hat{x}'], \quad (55)$$

where  $\kappa_a/\kappa_w = (1 - \alpha)(\tilde{y}/\tilde{n})/\tilde{w}$ . Because the ratio  $\kappa_a/\kappa_w$  is very close to unity, hiring effectively depends on the expected evolution of the labor share. Movements in the hiring rate then precipitate associated movements in unemployment and labor market tightness.

What is key for our purposes is that we have a model that, in conjunction with our contracting structure, produces plausible cyclical movements in the labor share and the relevant labor market quantities. Table 2 shows that our baseline meets this standard. With an empirically reasonable benchmark, it is then credible to study the importance of our contracting structure by replacing it with conventional Nash bargaining and then assessing the degree to which performance deteriorates.

The broad insights are likely to carry over to models that allow for alternative shocks and possibly alternative propagation mechanisms, as long as the baseline model with our contracting structure can generate empirically reasonable movements in the labor market variables.<sup>25</sup> In Gertler, Sala, and Trigari (2008), we verify this conjecture by embedding our framework of unemployment with staggered wage contracting within a contemporary quantitative macroeconomic model, of the type popularized by Christiano et al. (2005) and Smets and Wouters (2007).<sup>26</sup> The framework allows for multiple sources of business cycle dynamics as well as a rich set of endogenous model dynamics. In addition, we estimate the key model parameters using Bayesian methods. The estimated model indeed captures well the behavior of the labor market variables including the labor share. Replacing our wage contracting structure with period-by-period Nash bargaining then leads to a deterioration in performance similar to what we observed in the simple framework of this paper.

<sup>25</sup> Models that feature shocks other than total productivity typically capture realistic movements in labor productivity in part by allowing for variable intensity of capital utilization (e.g., Christiano et al. 2005; Smets and Wouters 2007).

<sup>26</sup> Christiano, Motto, and Rostagno (2008) similarly embed our labor market structure within a contemporary quantitative macro model. See Walsh (2005) and Trigari (2009) for early attempts to incorporate unemployment into a macroeconomic framework with nominal rigidities.

## VI. Conclusion

We have modified the Mortensen and Pissarides model of unemployment dynamics to allow for staggered multiperiod wage contracting. What emerges is a tractable relation for wage dynamics that is a natural generalization of the period-by-period Nash bargaining outcome in the conventional formulation. An interesting side product is the emergence of spillover effects of aggregate wages that influence the bargaining process. We then show that a reasonable calibration of the model can account reasonably well for the cyclical behavior of wages and labor market activity observed in the data. The spillover effects turn out to be important in this regard.

We also presented some evidence to suggest that the high relative cyclicity of new hires' wages may reflect cyclical composition effects as opposed to differences of wage flexibility within a firm between new and existing workers. That is, the evidence is consistent with procyclical movements in the average quality of jobs for new hires. In work in progress we are attempting to incorporate these compositional effects within our framework. We conjecture that as long as both high- and low-quality employment are procyclical with flexible wages, then the effects of our contracting framework on employment volatility will carry through to this framework.

## Appendix A

### Bargaining Set and Global Optimum

We first establish that the bargaining set is not convex, making the problem nonstandard. We then show, however, that among the set of deterministic wage contracts, we have a unique global optimum. The extended appendix in the working paper version of this paper, finally, shows that while there may be gains from having a lottery over different wage contracts, these gains are quite tiny and are easily offset by small transactions costs.

#### *Bargaining Set*

Let the function  $J(w, \mathbf{s}) = F(H(w, \mathbf{s}))$  describe the frontier of the bargaining set, where  $H(w, \mathbf{s})$  is worker surplus and  $J(w, \mathbf{s})$  is firm surplus. Given that next period's wage  $w'$  equals this period's wage  $w$  with probability  $\lambda$  and equals next period's target wage  $w^{*'}$  with probability  $1 - \lambda$ , we can write

$$\begin{aligned} H(w, \mathbf{s}) = & w - b + \beta E[\Lambda(\mathbf{s}, \mathbf{s}')][\rho\lambda H(w, \mathbf{s}') \\ & + \rho(1 - \lambda)H(w^{*'}, \mathbf{s}') - p\bar{H}_x(\mathbf{s}')]\mathbf{s} \end{aligned} \quad (\text{A1})$$

and

$$J(w, \mathbf{s}) = a - w - \frac{\kappa}{2}x^2(w, \mathbf{s}) + [\rho + x(w, \mathbf{s})]\beta E\{\Lambda(\mathbf{s}, \mathbf{s}')[\lambda J(w, \mathbf{s}') + (1 - \lambda)J(w^*, \mathbf{s}')]\mid \mathbf{s}\}, \quad (\text{A2})$$

with

$$\kappa x(w, \mathbf{s}) = \beta E\{\Lambda(\mathbf{s}, \mathbf{s}')[\lambda J(w, \mathbf{s}') + (1 - \lambda)J(w^*, \mathbf{s}')]\mid \mathbf{s}\}. \quad (\text{A3})$$

The slope of the bargaining set frontier is then given by

$$\frac{\partial J(w, \mathbf{s})}{\partial w} \bigg/ \frac{\partial H(w, \mathbf{s})}{\partial w} = -\frac{\mu(w, \mathbf{s})}{\epsilon(\mathbf{s})} < 0, \quad (\text{A4})$$

where

$$\epsilon(\mathbf{s}) = 1 + (\rho\lambda\beta)E\{\Lambda(\mathbf{s}, \mathbf{s}')\epsilon(\mathbf{s}')\mid \mathbf{s}\} > 0 \quad (\text{A5})$$

and

$$\mu(w, \mathbf{s}) = 1 + [\rho + x(w, \mathbf{s})](\lambda\beta)E\{\Lambda(\mathbf{s}, \mathbf{s}')\mu(w, \mathbf{s}')\mid \mathbf{s}\} > 0. \quad (\text{A6})$$

Let  $\mu_w(w, \mathbf{s}) \equiv \partial\mu(w, \mathbf{s})/\partial w$  and  $x_w(w, \mathbf{s}) \equiv \partial x(w, \mathbf{s})/\partial w$ . Then we can express the change in the slope as

$$-\frac{\mu_w(w, \mathbf{s})}{\epsilon^2(\mathbf{s})} > 0, \quad (\text{A7})$$

with

$$\begin{aligned} \mu_w(w, \mathbf{s}) &= x_w(w, \mathbf{s})(\lambda\beta)E\{\Lambda(\mathbf{s}, \mathbf{s}')\mu(w, \mathbf{s}')\mid \mathbf{s}\} \\ &+ [\rho + x(w, \mathbf{s})](\lambda\beta)E\{\Lambda(\mathbf{s}, \mathbf{s}')\mu_w(w, \mathbf{s}')\mid \mathbf{s}\} < 0, \end{aligned} \quad (\text{A8})$$

where

$$x_w(w, \mathbf{s}) = -\left(\frac{\beta\lambda}{\kappa}\right)E\{\Lambda(\mathbf{s}, \mathbf{s}')\mu(w, \mathbf{s}')\mid \mathbf{s}\} < 0. \quad (\text{A9})$$

Since  $-\mu_w(w, \mathbf{s}) > 0$ , it follows that the function  $J = F(H)$  is strictly decreasing and strictly convex, implying that the bargaining set is nonconvex. This contrasts with the conventional case of period-by-period Nash bargaining, where  $\mu_w(w, \mathbf{s}) = 0$ . In this instance the bargaining set is convex since the change in the slope of the bargaining set is zero. The negative impact of  $w$  on  $\mu$  arises in our case ultimately as a result of the “horizon effect” on bargaining in the multiperiod contract case that we described in the text. In particular, unlike the worker, the firm cares about the implications of the wage contract for new hires that come in subsequent periods. This means that how the firm discounts the future depends on the hiring rate  $x(w, \mathbf{s})$ , which in turn depends on  $w$ . An increase in  $w$  thus not only decreases the firm’s flow profits per worker but also reduces the hiring rate, implying lower expected profits in the future (since new hires coming in on this contract receive the higher wage). The decline in  $x(w, \mathbf{s})$  in

turn reduces  $\mu(w, \mathbf{s})$ . This effect, which is not present in the standard case, is responsible for the nonconvexity of the bargaining set in the multiperiod case.

As we show below, however, the departure from convexity is quantitatively small. This should not be surprising because as we showed in the text, the horizon effect that underlies the nonconvexity is trivial quantitatively under our baseline calibration. Further, as we show below, not only is the departure from convexity small, the bargaining frontier is well behaved: It is strictly decreasing and strictly convex and there are no kinks; that is, the bargaining set is smoothly nonconvex. Thus, as we show next, excluding lotteries, our Nash bargaining solution yields a unique global optimum.

### *Global Optimum*

We begin by establishing that the first-order condition defines an optimum when the economy is within a local region of the steady state and then show that the optimum is unique.

We may express the first-order condition for Nash bargaining as requiring that the slope of the bargaining set frontier equals the slope of the Nash product level sets:

$$-\frac{\mu(w, \mathbf{s})}{\epsilon(\mathbf{s})} = -\frac{\eta}{1-\eta} \frac{J(w, \mathbf{s})}{H(w, \mathbf{s})}. \quad (\text{A10})$$

Similarly, the second-order condition for a maximum requires that the change in the slope of the bargaining set frontier is smaller than the change in the slope of the Nash product level sets:

$$-\frac{\mu_w(w, \mathbf{s})}{\epsilon^2(\mathbf{s})} < \frac{\eta}{1-\eta} \frac{1}{H(w, \mathbf{s})} \left[ \frac{\mu(w, \mathbf{s})}{\epsilon(\mathbf{s})} + \frac{J(w, \mathbf{s})}{H(w, \mathbf{s})} \right]. \quad (\text{A11})$$

As we noted earlier, in the standard case of period-by-period Nash bargaining,  $\mu_w(w, \mathbf{s}) = 0$ . This implies that the second-order condition holds globally in this instance since the right side of the above expression is always positive. That the second-order condition holds globally implies that the first-order condition defines a unique optimum.

Because  $\mu_w(w, \mathbf{s}) > 0$  in our case, we cannot establish analytically that the second-order condition always holds. However, we show numerically that under our baseline calibration, the second-order condition is easily satisfied globally, making the optimum unique.

We begin by establishing that in the steady state the first-order necessary condition for our bargaining problem defines a unique optimum. Combining equations (A10) and (A11) implies that for the first-order condition to define a local optimum, the following condition must hold:

$$-\eta \frac{\mu_w(w, \mathbf{s}) J(w, \mathbf{s})}{\mu^2(w, \mathbf{s})} < 1. \quad (\text{A12})$$

Under our baseline calibration this condition is easily satisfied in the steady state:

$$-\left. \frac{\eta\mu_w(w, \mathbf{s})J(w, \mathbf{s})}{\mu^2(w, \mathbf{s})} \right|_{ss} = \frac{\eta(1-\rho)\beta\lambda^2}{1-\beta\lambda} = 0.1208 < 1. \quad (\text{A13})$$

Thus at the steady state the first-order condition defines an optimum. We also know from the text that the steady-state equilibrium is unique. Thus in the steady state, the first-order condition for Nash bargaining defines a unique global optimum.

We next show that we have a unique global maximum for the bargaining problem, even if we do not impose that the individual firm and its workers are in the steady state. That is, we ask whether an individual firm and its workers might have the incentive to deviate from the steady-state Nash bargaining solution, assuming that all other firms and workers remain at the steady state. We show that the answer is no.

In particular, suppose that a firm and its workers are contemplating for one contracting spell a wage that is different from the steady-state wage. Is there an alternative wage within the bargaining set that satisfies the first-order conditions that they may prefer? By constructing the relevant bargaining set, we can show numerically that no other solution exists.

Let  $w$  denote a wage within the bargaining set  $[\underline{w}, \bar{w}]$  that two bargaining parties are contemplating and, as in the text, let  $\tilde{H}$  and  $\tilde{J}$  denote steady-state worker and firm surplus, respectively. Then in this case, the worker and firm surplus are given by

$$H(w) = w - b + \beta[\rho\lambda H(w) + \rho(1-\lambda)\tilde{H} - \tilde{p}\tilde{H}] \quad (\text{A14})$$

and

$$J(w) = \tilde{a} - w - \frac{\kappa}{2}x^2(w) + [\rho + x(w)]\beta[\lambda J(w) + (1-\lambda)\tilde{J}], \quad (\text{A15})$$

with

$$\kappa x(w) = \beta[\lambda J(w) + (1-\lambda)\tilde{J}]. \quad (\text{A16})$$

In addition, the boundaries of the bargaining set are defined by

$$H(\underline{w}) = 0 \quad (\text{A17})$$

and

$$J(\bar{w}) = 0. \quad (\text{A18})$$

We can next solve the system given by (A14)–(A18) to first construct the bargaining set and then show that the second-order condition given by equation (A11) holds globally for this case. Figure A1 portrays the numerical solution for the bargaining set along with the Nash product level sets. There is a unique optimum in which the slopes of the two sets are equal. At any other point, the change in the slope of the bargaining set is smaller than that for the Nash

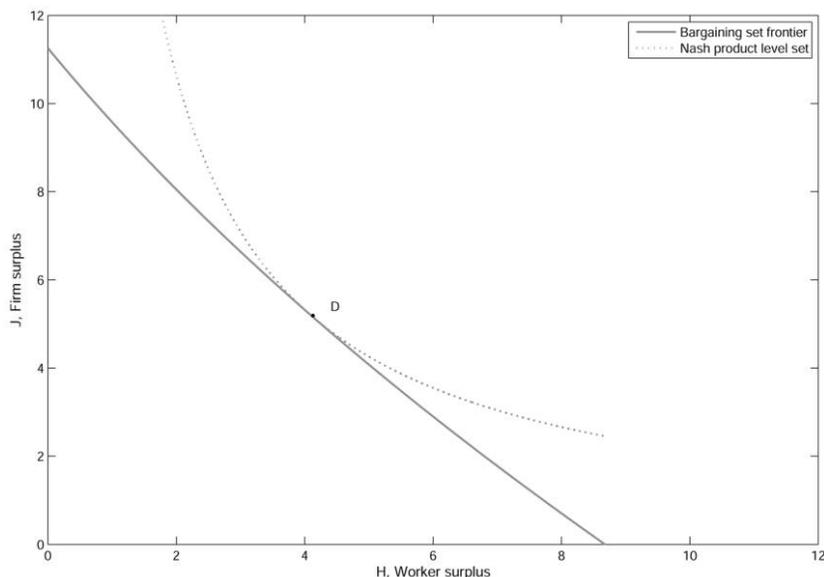


FIG. A1.—Global optimum, deterministic wage contract

product level set. Hence, the first-order condition defines a unique global optimum. It is also instructive to observe that even though the bargaining set is not convex, the departure from convexity is both smooth and quantitatively small. This is what ensures that the optimum is unique.

What we have shown thus is that, given that the rest of the economy is in the unique steady state, any individual firm and its workers have no incentive to deviate for one contracting spell from the steady-state solution. For simplicity we have assumed that the firm and its workers revert to the steady-state solution once the contract has elapsed. All we need to assume, however, is that the two parties expect to be back at the unique steady state at some point in the future. We can use a recursive argument to establish uniqueness, along the lines we have just presented. In particular, we consider that point in the future that is the last contract before the firm and its workers revert to the steady state. We then show that no alternative is preferable to the steady state, using the exact same argument we have just employed. We can then work all the way back to the present, showing that the firm and its workers have no incentive to deviate today.

Thus we have proved that there is a unique global optimum for the individual firm when the rest of the economy is in the steady state. It is straightforward to verify numerically that the uniqueness result holds when the aggregate economy exhibits small percentage fluctuations around the steady state of the type we analyzed in the text.

We have shown that despite the nonconvex bargaining set, the bargaining problem we posed has a unique global optimum over the space of deterministic

wage contracts. However, the nonconvex bargaining set opens up the possibility that a lottery contract that randomizes wages might dominate a deterministic one. However, because the bargaining set is very close to being linear (see fig. A1), the gains from a lottery are negligible and easily offset by small transactions costs of running the lottery. We show this explicitly in the extended appendix available at the authors' Web sites.

## Appendix B

### Complete Log-Linear Model

We next present the complete log-linear model. A tilde denotes the steady-state value of a variable and a hat the log deviation from the steady-state value. To simplify notation we use time indexes, and for any variable  $z$ , we write  $\hat{z}(\tilde{w}, \mathbf{s})$  as  $\hat{z}_t$ .

Technology:

$$\hat{y}_t = \hat{z}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t. \quad (\text{B1})$$

Resource constraint:

$$\hat{y}_t = y_c \hat{c}_t + y_i \hat{i}_t + (1 - y_c - y_i)(2\hat{x}_t + \hat{n}_t), \quad (\text{B2})$$

where  $y_c = \tilde{c}/\tilde{y}$  and  $y_i = \tilde{i}/\tilde{y}$ .

Matching:

$$\hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t. \quad (\text{B3})$$

Employment dynamics:

$$\hat{n}_{t+1} = \hat{n}_t + (1 - \rho) \hat{x}_t. \quad (\text{B4})$$

Transition probabilities:

$$\hat{q}_t = \hat{m}_t - \hat{v}_t, \quad (\text{B5})$$

$$\hat{p}_t = \hat{m}_t - \hat{u}_t. \quad (\text{B6})$$

Unemployment:

$$\hat{u}_t = -(\tilde{n}/\tilde{u}) \hat{n}_t. \quad (\text{B7})$$

Capital dynamics:

$$\hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \delta \hat{i}_t. \quad (\text{B8})$$

Aggregate vacancies:

$$\hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_t. \quad (\text{B9})$$

Consumption and savings:

$$0 = E_t \hat{\Lambda}_{t,t+1} + [1 - \beta(1 - \delta)] E_t \hat{r}_{t+1}. \quad (\text{B10})$$

Marginal utility:

$$E_t \hat{\Lambda}_{t,t+1} = \hat{c}_t - E_t \hat{c}_{t+1}. \quad (\text{B11})$$

Aggregate hiring:

$$\hat{x}_t = E_t \{ \boldsymbol{\kappa}_a \hat{a}_{t+1} - \boldsymbol{\kappa}_w \hat{w}_{t+1} + \hat{\Lambda}_{t,t+1} + \beta \hat{x}_{t+1} \}, \quad (\text{B12})$$

where  $\boldsymbol{\kappa}_a = \tilde{a}\boldsymbol{\kappa}$ ,  $\boldsymbol{\kappa}_w = \tilde{w}\boldsymbol{\kappa}$ , and  $\boldsymbol{\kappa} = \beta(\kappa\tilde{x})^{-1}$ .

Marginal product of labor:

$$\hat{a}_t = \hat{y}_t - \hat{n}_t. \quad (\text{B13})$$

Capital renting:

$$\hat{r}_t = \hat{y}_t - \hat{k}_t. \quad (\text{B14})$$

Effective bargaining power:

$$\hat{\chi}_t = -(1 - \tilde{\chi})(\hat{\mu}_t - \hat{\epsilon}_t), \quad (\text{B15})$$

with

$$\hat{\epsilon}_t = \rho\lambda\beta E_t(\hat{\Lambda}_{t,t+1} + \hat{\epsilon}_{t+1}) \quad (\text{B16})$$

and

$$\hat{\mu}_t = (\tilde{x}\lambda\beta)\hat{x}_t - (\tilde{x}\beta\lambda)(\boldsymbol{\kappa}_w\lambda\tilde{\mu})(\beta\lambda)\tilde{\mu}E(\hat{w}_t - \hat{w}_{t+1}) + \lambda\beta E_t(\hat{\Lambda}_{t,t+1} + \hat{\mu}_{t+1}). \quad (\text{B17})$$

Spillover-free target wage:

$$\hat{w}_t^o = \tilde{\chi}\varphi_a\hat{a}_t + (1 - \tilde{\chi})\varphi_p\hat{p}_t + [\tilde{\chi}\varphi_x + (1 - \tilde{\chi})\varphi_p]\hat{x}_t + \varphi_x[\hat{\chi}_t - \beta(\rho - p)E_t\hat{\chi}_{t+1}], \quad (\text{B18})$$

where

$$\varphi_a = \tilde{a}\tilde{w}^{-1}, \quad \varphi_x = \kappa\tilde{x}^2\tilde{w}^{-1}, \quad \varphi_p = \tilde{p}\tilde{\beta}\tilde{H}\tilde{w}^{-1}, \quad \varphi_x = \tilde{\chi}(1 - \tilde{\chi})^{-1}\kappa\tilde{x}\beta^{-1}\tilde{w}^{-1}.$$

Aggregate wages:

$$\hat{w}_t = \gamma_o\hat{w}_{t-1} + \gamma_o\hat{w}_t^o + \gamma_f E_t\hat{w}_{t+1}, \quad (\text{B19})$$

where

$$\gamma_o = (1 + \tau_2)\phi^{-1}, \quad \gamma_o = \mathbf{s}\phi^{-1}, \quad \gamma_f = (\tau\lambda^{-1} - \tau_1)\phi^{-1},$$

$$\phi = (1 + \tau_2) + \varsigma + (\tau\lambda^{-1} - \tau_1), \quad \varsigma = (1 - \lambda)(1 - \tau)\lambda^{-1},$$

$$\tau = \frac{(\rho\beta\lambda)\psi}{1 + (\rho\beta\lambda)\psi}, \quad \psi = \tilde{\chi}\tilde{\mu} + (1 - \tilde{\chi})\tilde{\epsilon},$$

$$\tau_1 = \tilde{\chi}\tilde{\mu}\beta[(\rho - \tilde{p})(\tilde{\chi}\beta\lambda)(\lambda\tilde{\mu}) + \tilde{p}\eta^{-1}](1 - \tau),$$

$$\tau_2 = \tilde{\chi}\tilde{\mu}(\tilde{\chi}\beta\lambda)(1 - \lambda\tilde{\mu})(1 - \tau).$$

Technology process:

$$\hat{z}_i = \rho_z \hat{z}_{i-1} + \epsilon_i^z. \quad (\text{B20})$$

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