Intergenerational Equity and the Investing of Rents from Exhaustible Resources

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Invest all profits or rents from exhaustible resources in reproducible capital such as machines. This injunction seems to solve the ethical problem of the current generation shortchanging future generations by "overconsuming" the current product, partly ascribable to current use of exhaustible resources.[1] Under such a program, the current generation converts exhaustible resources into machines and "lives off" current flows from machines and labor. Under such a program one might assume that in some sense the total stock of productive capital was never depleted since ultimately the exhaustible resource stock will be transmuted into a stock of machines and, given that machines are assumed not to depreciate, no stock either of machines or of exhaustible resources is ever consumed. If in this sense the stock of productive capital is not being depleted, what can one say about the time path of current output and current consumption per head? For the case of per capita consumption remaining constant over time, one could say that no generation was better off than another. Intergenerational equity was being achieved.[2] For simplicity, we shall assume *ZPG* or a constant population so we need only ask what happens to the time path of aggregate consumption. Let me restate the problem in brief: if society invests all rents from exhaustible resources in reproducible capital goods, and invests only this amount, i.e., consumes the remainder of the product given population constant, will consumption and output rise, remain constant, or fall over time?

I shall formally set this problem out below and solve it for the case of a Cobb-Douglas technology. The Cobb-Douglas technology has the important property that each input (in particular, the flow of minerals from an exhaustible resource) is essential for producing a positive output of the single produced commodity. Thus the economy cannot exhaust any natural resource and continue to have positive consumption and output. Beckmann (1974, 1975), Solow, and Solow and Wan have used the Cobb-Douglas technology in their analyses of utilization of exhaustible resources in aggregate dynamic models.

Production in the model at period t will be assumed to require inputs of reproducible capital $k(t)$, flows of mineral from an exhaustible resource $y(t)$ and labor. The labor force is constant so we can set it at one unit. The $k(t)$, $y(t)$, commodity output $x(t)$, and consumption $c(t)$ are defined in per capita terms. The technology $f(k(t), y(t), 1)$ will be assumed to exhibit constant returns to scale so that $f(\cdot)$ is homoge-

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[1]The idea for this paper arose after hearing a seminar by Anthony Scott on resource policy. He estimated the returns Canadians might receive in 1975 if they had invested all resource royalties in assets yielding the current rates of interest prevailing since 1911. The interest each year was to be consumed. He labeled such a strategy as a "Saudi Arabian" program!

[2]See Kenneth Arrow for a systematic exploration of savings rules and intergenerational equity within the context of a model of accumulation of reproducible capital in the absence of exhaustible resources. The current interest in intergenerational equity was aroused by remarks of John Rawls (sec. 44). One should of course consult Rawls, Arrow, and Robert Solow for an introduction to the diverse notions of intergenerational equity which have been proposed for consideration in the current investigations. Rawls was concerned with the problem of balancing the relative burden of savings on early generations with the burden on later generations. Capital was being *accumulated* over some part of a society's program of consumption and investment. With exhaustible resources, one must be concerned with *forestalling decumulation* of society's productive capital in order to achieve some notion of intergenerational equity.

neous of degree one. The value of $x(t) = f(\cdot)$ will be zero if any argument of $f(\cdot)$ is zero. That is, each input is essential. The marginal productivities $\partial f/\partial k$ and $\partial f/\partial y$ are assumed to be positive; $\partial^2 f/\partial k^2$ and $\partial^2 f/\partial y^2$ are assumed to be negative. Let $f_k \triangleq \partial f/\partial k$, $f_y \triangleq \partial f/\partial y$, $f_{kk} \triangleq \partial^2 f/\partial k^2$, $f_{ky} \triangleq \partial^2 f/\partial k \partial y$, and $f_{yy} \triangleq \partial^2 f/\partial y^2$. A D before a variable will indicate the time derivative of that variable (for example, $Dk \triangleq dk/dt$). At any instant of time, the product $x(t)$ is completely divided between current consumption $c(t)$, investment Dk and extraction costs $ay(t)$, where a is the cost measured in units of the single produced commodity of extracting one unit of the exhaustible resource. Thus, we have our accounting relation

$$x(t) = c(t) + Dk + ay(t)$$

Our savings or investment function is

$$(1) \qquad Dk = (f_y - a)\, y(t)$$

Efficiency of exhaustible resource extraction requires that the rate of return from a unit of reproducible capital equal the rate of return from owning a unit of deposits of the exhaustible resource.[3] In price terms, this condition is characterized by the current capital gain on mineral deposits being equal to the interest rate or rate of return on reproducible capital. In our one-commodity world, this condition is satisfied by *the rate of change in the marginal product of the mineral being equal to the marginal product of reproducible capital.* This is sometimes referred to as the Hotelling Rule. It characterizes the efficient exploitation of an exhaustible resource. That is

$$(2) \qquad \frac{d \log (f_y - a)}{dt} = f_k$$

or

$$(2') \qquad f_{yy} Dy + f_{yk} Dk = f_k (f_y - a)$$

Relations (1) and (2) define the dynamics of the economy. There are two differential equations in the variables $y(t)$ and $k(t)$. We require initial values $k(0)$ and $y(0)$ in order to define the time paths of $y(t)$ and $k(t)$. We shall assume that $k(0)$ and $y(0)$ are selected so that the initial stock of exhaustible resource S is precisely sufficient to sustain the economy over infinite time. We shall remark below that there exists a finite S which will yield the consumption path below. By definition, $dS/dt = -y(t)$, where the stock S is defined in per capita terms.

Aggregate output is rising, constant, or falling over an interval of time as $Dx \gtreqless 0$. Now from the definition of the production function, we get

$$(3) \qquad Dx = f_k Dk + f_y Dy$$

For the case of the Cobb-Douglas technology, we have

$$x = k^{\alpha} y^{\beta} 1^{\gamma}$$

with $\alpha + \beta = 1$ and $f_k \triangleq \alpha x/k$ and $f_y \triangleq \beta x/y$. Also $f_{yy} \triangleq \beta x(\beta - 1)/y^2$ and $f_{yk} \triangleq \alpha\beta x/yk$.

For the case of the Cobb-Douglas technology, (2′) becomes

$$f_y Dy - xDy/y + f_k Dk = (y/\beta) f_k (f_y - a)$$

and substituting for Dk from (1), we get

$$(4) \qquad \beta[f_y Dy + f_k(f_y - a) y] = f_y Dy + f_k (f_y - a) y$$

Since $0 < \beta < 1$, equation (4) can only be satisfied if $f_y Dy + f_k(f_y - a)y = 0$ but $f_y Dy + f_k(f_y - a)y$ is the right-hand side of (3). Thus we have established that x will be constant over time and since $c(t) = (1 - \beta)x(t)$, (recall $f_y y = \beta x$), we have the result that consumption per head will be constant over time. Given the finiteness of natural resource stock, it will be necessary over infinite time to have the current flow of resources extracted asymptotically approach zero as time tends to infinity. By Solow's definition of intergenerational equity—namely per capita consumption remaining constant over time—we have established that the savings investment rule (invest all net returns from exhaustible re-

[3]Since in a well-behaved problem (e.g., the case with a Cobb-Douglas production function) f_y always increases as $t \to \infty$, one has only to assure that the extraction costs are such that $(f_y - a) > 0$ at t_0.

sources in reproducible capital) implies intergenerational equity. A perusal of the mathematics of Solow's paper indicates that this result was implicit in his mathematics—to preserve $Dc = 0$, society should invest the current returns from the utilization of flows from the stock of exhaustible resources.

We have in fact obtained the rule for a model with nonzero extraction costs. Solow had no extraction costs in his formulation. He proved that the existence of a solution required that $\beta < \alpha$. We take this as a necessary condition for existence. It implies that the share of output ascribable to natural resources be less than that share ascribable to reproducible capital—a condition which empirical results indicate is unambiguously satisfied. To be precise, the only model in which the existence of a solution with c positive over infinite time and S finite has been established is the above Cobb-Douglas case with extraction costs set at zero. On reading the above note Solow pointed out that the rule "invest exhaustible resource rents and c will remain constant" is very general. To see this substitute from (1) and (2) in the relation $Dx = f_k Dk + f_y Dy$ for f_k and Dk to get

$$Dx = \left\{ \frac{df_y}{dt} \Big/ (f_y - a) \right\} (f_y - a) y + f_y Dy$$

$$= \frac{d(f_y y)}{dt} = \frac{d(Dk + ay)}{dt}$$

Given $x = c + Dk + ay$ we conclude that $Dc = 0$ regardless of whether $Dx = 0$. Thus we have established, for general technologies, the rule: "the investment of current exhaustible resource returns in reproducible capital implies per capita consumption constant." For the Cobb-Douglas case $Dk + ay = f_y y = \beta x$. Thus from above we have $Dx = \beta Dx$ and since $\beta \neq 1$, $Dx = 0$. If there is depreciation of reproducible capital at the rate δ per unit capital per unit time, then net capital accumulation is currently of the amount $dk/dt + \delta k(t)$ and given our savings rule, equation (1) becomes $Dk + \delta k = (f_y - a) y(t)$. Reworking the steps for solving for Dc above reveals that $Dc = -\delta f_k k$. Hence our savings investment rule will not provide for the maintaining of per capita consumption constant over time. The current decline in per capita consumption is simply the amount of the produced commodity required to offset the current amount of depreciation in the reproducible capital. Arrow's results would not turn on whether he has reproducible capital depreciate; he does not explicitly treat depreciation. In my 1976 paper the present model is extended to cover cases of many exhaustible resources. The intergenerational equity result has also been established in a Uzawa two-sector model with an exhaustible resource.

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