Ideology, Tactics, and Efficiency in Redistributive Politics
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We model the electoral politics of redistribution when voters and parties care about inequality in addition to their private concerns for consumption and votes, respectively. Ideological concerns about income redistribution lead each party to adopt a general proportional income tax, adjusted to appeal to the ideological leanings of high “clout” groups, with disproportionately many “swing” voters, which the parties also ply with pork-barrel projects. Our results relate to “Director’s Law,” which says that redistributive politics favors middle classes at the expense of both rich and poor.

I. INTRODUCTION

Normative analyses of taxation focus on the optimal balance between equity and efficiency according to some postulated social welfare function. Informal discussions of this trade-off have a long history, but formal modeling derives from the pioneering work of Mirrlees [1971] on income taxation, and Diamond and Mirrlees [1971] on commodity taxation. Atkinson and Stiglitz [1980] review this literature. However, the recognition that actual policies are the outcome of a political process, and that they depart widely from the normative prescriptions, has led to much research in the positive mode.

The prevailing positive models of redistribution assume that all players in the politico-economic game are selfishly motivated: citizens and voters care for private consumption; and politicians for power, reelection, or material gains. In reality, citizens and politicians alike do care for distributive equity as a public good. They may not sacrifice much private consumption to achieve greater equity on an individual basis because of free rider problems, but they may vote to implement general policies that will require them and everyone else to make some such sacrifice. Therefore, these ideological aspects of voters’ choices and parties’ platforms play an important role in the political process. In this paper we study how the ideological motive for redistribution

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mixes with the tactical and political motives to determine tax and transfer policies.

Positive political theory of taxes and transfers has been based on two kinds of electoral models. On the one hand, there are models of voting over income tax schedules: for example, Romer [1975], Roberts [1977], and Roell [1995]. In these, each voter is concerned only with his or her own economic well-being, and can calculate how any given tax scheme will affect this, taking into account the direct effects as well as the indirect ones working through the general equilibrium of the whole economy. Models in this family typically impose a linear tax scheme—the proceeds of which must be divided equally among all citizens. The parties are concerned solely with their vote, and in equilibrium the utility of the median voter gets maximized.

A second family of models focuses on pork-barrel politics: for example, Cox and McCubbins [1986], Lindbeck and Weibull [1987], Myerson [1993], and Dixit and Londregan [1995, 1996]. In these, vote-maximizing parties promise tactical transfers to high "clout" electoral groups who are most likely to shift their votes in response to these inducements of private consumption. The pork-barrel models share a recognition that voters have attachments to the political parties which can be loosened through the offer of private consumption in the form of pork-barrel spending. In some cases the nature of the allegiance is left in the background. In others, such as Lindbeck and Weibull and the Dixit and Londregan papers, it is modeled as stemming from voters' differing affinities for ideological positions with which the parties are associated but which they cannot change during the course of a single election. While the literature makes progress toward recognizing that the parties and the voters care about the overall shape of public policy as well as their own well-being, the models that recognize that voters make these calculations do not permit political parties to modify their ideological positions. Nor are the ideological positions related to the distribution of income. Instead, the Lindbeck-Weibull and Londregan-Dixit models require that when voters care about ideological aspects of public policy beyond their own private consumption, it is about issues that are independent of the distribution of income, while the remaining models of the income tax schedule and of pork-barrel politics do not explic-

1. There are also lobbying models of pork-barrel politics that focus on bribes paid by lobbyists to influence elected officials, most prominently Grossman and Helpman [1994], but they are not our concern here.
Itly model voters' concerns for public policy at all. This is particularly troublesome because income redistribution is often a central theme in parties' ideologies, and raises the question: "will the prediction of the pork-barrel models that political parties will offer special redistributive treatment to individuals from high "clout" groups survive when ideologies matter to the voters and the political parties and relate directly to the level of income inequality?"

In this paper we remove the separation between ideology and income redistribution. Just as voters care about the implications of income transfers for their own well-being, so do they also genuinely care about income inequality. This concern has fundamental implications for pork-barrel politics because it attaches to the very instruments politicians use to appeal to voters' self-interest. That voters' equity concerns are not mere rhetorical masks for self-interest is clear. There are rich liberals who will vote for a leftist party that promises a high tax rate, and poor libertarians who will support a rightist party even though they will personally benefit little from its tax and transfer policies.

We also recognize that political parties care about more than their performance at the ballot box. As with the voters so do the parties genuinely care about social issues such as equity, sometimes clinging to their ideologies even when these result in humiliating election losses. The British Labour party lost four successive general elections from 1979 to 1992 by sticking to an increasingly unpopular leftist ideology before it finally changed its platform. In the United States, the Democrats in the 1980s, and perhaps the Republicans in the 1990s, provide other examples of adherence to extreme ideologies even at the cost of electoral success.

We construct a model in which voters balance their private consumption benefits against their concern for social welfare, and parties similarly weigh the trade-off between votes and their own ideologies.

We find that the parties use different tax instruments to achieve different goals. Although they could set distinct marginal tax rates for every group, in equilibrium they choose not to. Instead, each party balances its own ideological concerns about taxation against electoral pressures for an equity efficiency trade-off that reflects the preferences of pivotal voters in the electorate. The result is that each party offers its own uniform marginal tax rate. In equilibrium this tax policy is augmented by lump-sum...
group-specific transfers that depend on how responsive groups will be to blandishments of tactical income transfers aimed at increasing the private consumption of group members.

We also make some contact with the normative literature on equity and efficiency. As one would expect, we find that when the deadweight losses caused by taxation are incorporated into the model, the political parties become more abstemious about both ideological and tactical income redistribution.

The following section sets forth our model in detail, while Section III derives the implications of each party's optimal tax and transfer policies. This sheds new light on "Director's Law," which says that middle classes succeed in redistributive politics at the expense of both the rich and the poor. In Section IV we solve the general equilibrium for a special case of the model. A brief Section V provides concluding remarks.

II. The Model

The general structure of the model is based on Lindbeck and Weibull [1987], Cox and McCubbins [1986], and Dixit and Londregan [1996]. Two parties compete for votes. Each individual cares about two things: an ideological issue, and his/her private consumption. Then each party is characterized by its most preferred ideological position, and its program of redistributive taxes and transfers which determines individuals' consumption quantities. The earlier models treated the parties' most preferred ideological positions as fixed within the time frame of a single election, but left them free to adjust taxes and transfers to gain electoral favor. Here we impose no such constraint, and the parties are free to modify their redistributive policies in ways that increase or decrease income inequality as well as to shift resources toward electorally pivotal groups of voters.

Each group is an identifiable economic interest. Each party can target some groups to receive pork-barrel benefits, and others to pay taxes to finance these benefits. There is considerable ideological heterogeneity within each group, and in response to the two parties' policy platforms, each group's membership splits into those who favor the one party or the other. At the margin, the parties can adjust their redistributive policies to win more voters in a group. But there is a budget constraint; therefore greater favors to some groups entail smaller favors to other groups, and a
loss of votes at the margins in those groups. In equilibrium each party's policy balances these gains and losses of votes per budget dollar across groups. As a result, the groups that are rich in votes at the margin are the gainers from the political process. Groups that are more numerous in inframarginal votes do not benefit: the party they favor takes them for granted, and the other party writes them off.

We now lay out the model, explaining the components in greater detail.

A. Voters and Groups

The electorate consists of groups labeled \( i = 1, 2, \ldots, g \). The groups are distinguished by some observable characteristics on which taxes or transfers can be based. These can include location, occupation, or income. Since we allow only a finite number of groups, we confine income to a finite number of categories instead of being a continuous variable, but since the number of groups can be arbitrarily large, this seems harmless.

The proportion of population that is in group \( i \) will be denoted by \( N_i \). Thus, the total population is normalized to unity:

\[
\sum_i N_i = 1.
\]

The \( N_i \) are given exogenously. The potential productivity of each member of group \( i \) is written \( Y_i \), which would equal his or her earnings or gross income in the absence of any taxes or transfers. The consumption (or net or disposable income) after taxes and transfers is written \( C_i \). We write \( C \) for the \( g \)-dimensional vector with components \( (C_i | i = 1, 2, \ldots, g) \). Similarly for \( Y \), and for vectors of other entities that will appear later.

B. Budget Constraint

We take the productivities \( Y_i \) to be exogenous. However, for most of the analysis we allow for deadweight losses in the tax process. In this respect we generalize the models of Cox and McCubbins [1986], Lindbeck and Weibull [1987], and Dixit and Londregan [1996]. We do this by imposing the constraint,

\[
\sum_i N_i C_i \leq \sum_i N_i Y_i - \frac{1}{2} \delta \sum_i N_i (C_i - Y_i)^2,
\]
or

\[ H(C) = \sum_i N_i C_i - \sum_i N_i Y_i + \frac{1}{2} \delta \sum_i N_i (C_i - Y_i)^2 \leq 0. \]

Thus, any attempt to achieve consumption quantities different from people's productivities reduces the total amount available for consumption. The parameter \( \delta > 0 \) measures the importance of these economic distortions.

The constraint (1) can be thought of as a "social budget constraint." It is a reduced form in that it does not specify the precise mechanics of how taxes and transfers generate distortions. To be more precise, using the jargon of macroeconomic policy analysis, \( \delta \) is not a "deep parameter." Our focus is more on the ideology of redistribution as it is embodied in the voters' and parties' objectives. Therefore, we deemed it necessary to simplify the specification of the economics of the model in order to keep the analysis tractable. Also, it yields results with considerable intuitive appeal.

C. Ideology

We define social welfare to be a weighted average of two extreme or caricature ideologies. The purely rightist function is defined by

\[ S^R(C) = -\frac{1}{2} \sum_i N_i (C_i - Y_i)^2. \]

This can be interpreted in two ways. According to a political or philosophical interpretation, it captures the belief that individuals are entitled to the fruits of their own productivity, and any government policy that deviates from this standard is to be regarded as a loss (negative welfare level). An economic interpretation notes that constraint (1) will hold with equality, so we can substitute from it to get an alternative and equivalent expression for the social welfare; namely,

\[ S^R(C) = \left( \frac{1}{\delta} \right) \sum_i N_i (C_i - Y_i). \]

Since the \( Y_i \) are constants, this is tantamount to being concerned solely with the aggregate consumption, that is, valuing economic efficiency and being totally unconcerned about equity.
The purely leftist social welfare function is

\[ S^L(C) = -\frac{1}{2} \sum_i N_i (C_i - \hat{C})^2, \]

where

\[ \hat{C} = \sum_i N_i C_i \]

is the population average of consumption. This is a decreasing function of the variance of consumption (the factor \( \frac{1}{2} \) merely simplifies some later algebra). It expresses an egalitarian political philosophy: its best value, namely 0, is achieved only when all the \( C_i \) are equal. If there is perfect equality, then it does not matter how low the common consumption level \( \hat{C} \) is. Thus, this function also shows total unconcern for economic efficiency.

The actual social welfare function we associate with any voter or party is a weighted sum of these two, with weights of \( X \) on the rightist function and \( (1 - X) \) on the leftist:

\[ S(C, X) = -\frac{1}{2} X \sum_i N_i (C_i - Y_i)^2 - \frac{1}{2} (1 - X) \sum_i N_i (C_i - \hat{C})^2, \]

where \( 0 \leq X \leq 1 \). Each voter and each party will have a value of \( X \) in this range.\(^2\)

D. Heterogeneity within Groups

Within each group \( i \) we allow a whole spectrum of attitudes toward social welfare. The members of a given group are not all political “clones” with the same relative weights on equality and efficiency, as would be characterized by a single value of \( X \) for all members of a given group. Even among autoworkers at the same plant and partners in the same law firm, there are disagreements about politics. However, while the entire spectrum of political beliefs can be found within almost any identifiable group, the fraction of, say, political liberals varies considerably across groups. The cumulative distribution function, \( \Phi_i(X) \) that specifies the fraction of individuals at or to the left of any particular \( X \)-value, thus varies among groups. For any given \( X \) in the interval \((0,1)\), a proportion \( \Phi_i(X) \) of members of group \( i \) have social welfare

\(^2\) The range of \( X \) can be further restricted if the two extreme caricature social welfare functions are thought unlikely to exist in the pure forms.
functions to the left of $X$, placing a weight of less than $X$ on the rightist component.  

E. Heterogeneity among Groups

Our framework allows for differences among the political leanings of groups even as we recognize that within each group there are some individuals who do not share the dominant political outlook for their group. Just as our framework allows us to formalize one individual $a$ having political preferences to the “left” of individual $b$ as $X_a < X_b$, so too can we compare the political leanings of groups using their distribution functions. For example, if $\Phi_j(X)$ is a rightward shift of $\Phi_i(X)$ in the sense of first-order stochastic dominance, so that for all $X$ satisfying $0 < X < 1$,

$$\Phi_j(X) < \Phi_i(X),$$

then it makes sense to think of group $j$ as being to the “right” of group $i$.

One of our results below, namely that the political equilibrium will more closely reflect the ideology of the middle classes than that of either the very rich or the very poor will make use of the assumption that richer groups are more conservative than poorer groups in the sense that if $Y_i < Y_j$, $\Phi_j(X)$ is a rightward shift of $\Phi_i(X)$. This assumption finds empirical support in McClosky and Zaller [1984, pp. 154–155] and Moffitt, Ribar, and Wilhelm [1996, Figure 5]. However, we do not need the assumption for most of our analysis.

A member of group $i$ with social welfare parameter $X$ will account for both ideology and consumption when deciding how to cast his/her ballot, voting to maximize

$$\alpha_i C_i + S(C_i, X).$$

The parameter $\alpha_i$, assumed to be positive, measures the selfishness of members of group $i$. Just as groups differ in their ideological leanings, we allow for them to differ in the relative weight that their members accord to private consumption versus ideology. Those with a higher $\alpha_i$ will be swayed more easily by promises of private consumption even though this entails lower social welfare.

3. The associated probability density function is written as $\phi_i(X) = \Phi_i'(X)$. 

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F. Parties

There are two parties labeled L and R. Each has its own ideology about the distribution of income, reflected in weights $X_L$ and $X_R$ with $X_L < X_R$; the complete objectives of the parties will be specified shortly. We assume that the parties’ ideologies are fixed for the duration relevant for the analysis. This reflects the fact that parties find it very difficult because of their internal politics to make genuine shifts in ideological positions, or (in the United States) that they get locked into positions during the intraparty struggles in the primaries, or that they find it optimal for reputational reasons to adhere to a particular position. This does not prevent them from presenting political platforms that compromise their ideology for the sake of electoral advantage; the same will be true of the parties in our model.

The parties make promises of tax and transfer policies to the electorate. The ultimate effect of these policies appears in the form of the consumption quantities $C_i$, so we can summarize the parties’ strategies by their implied consumption vectors $C^L$ and $C^R$.

G. Cut-points

Given these strategies, the voters of each group will split into those who favor party L (those with low X) and those who favor R (those with high X). The cut-point $X_i$ for group $i$ is defined as the value of X that makes a voter from this group indifferent between the two parties. We postpone the derivation of the actual formula for $X_i$; for now we merely recognize that the $X_i$ will be functions of the parties’ platforms ($C^L$, $C^R$). Then in group $i$ the fraction $\Phi_i(X_i)$ of its membership that is located to the left of its cut-point votes for party L, and the rest vote for party R. Therefore, the vote shares of the two parties are

\begin{align*}
V^L(C^L, C^R) &= \sum_i N_i \Phi_i(X_i(C^L, C^R)), \\
V^R(C^L, C^R) &= \sum_i N_i [1 - \Phi_i(X_i(C^L, C^R))].
\end{align*}

The parties choose their tax and transfer strategies, or equivalently, the consumption vectors $C^L$ and $C^R$, bearing in mind the electorate’s choices as given by the cut-points $X_i$. We consider the Nash equilibrium of this game, where each party chooses its strategy to maximize its own objective, given the strategy of the other.
H. Parties' Objectives

There is no generally agreed-upon objective that political parties maximize. There are some prominent models: of these our formulation closely relates to Wittman [1983], Calvert [1985], and Roemer [1997].

In the Wittman-Calvert model parties care about the policies actually implemented, and about the perquisites of office. Each party's platform affects the outcome of the election and the eventual policy. Therefore, each makes a trade-off between its platform and electoral appeal. Roemer introduces a third aspect, namely a party's concern for its public position per se, even when this position has no realistic chance of being implemented into policy at the next electoral cycle, and even when the party suffers some electoral cost by adopting such a position. Roemer's third objective is closest to our focus on parties' ideologies.

The extreme "pro-life" position of the Republican party in 1996, or the British Labour Party's espousal of public ownership throughout the 1970s and 1980s, are stark examples of how parties sacrifice electoral success for ideological purity. Of course, such positions may have a reputational role, and the parties may believe that their current stance will influence policy over a longer horizon than a single election. But such calculations are not made explicit in any of these models, which focus on a single election.

In the Wittman-Calvert model, the parties care exclusively about results. The parties want to influence the content of postelection policy, and in some variants of the model, they also seek the intrinsic benefits of holding office. In practice, the political parties do not calculate their winning electoral margins to the last vote, compromising on policy just enough to win the last crucial vote needed for victory. Such calculations are hard enough to make on the floor of a legislature. In a mass election no politician stops campaigning when he senses he is at least one vote ahead of the competition. Instead, there are a number of reasons why politicians might want to maximize their vote totals. For one there is the uncertainty of victory. The larger is one's expected vote total, the more "bad luck" in the form of unanticipated scandal, campaign gaffs, and idiosyncratic voter reactions it takes to spoil the anticipated victory. And there are other reasons politicians may care about the magnitude of the margin of victory: for example, concern about longer run electoral dynamics. Building a larger margin today increases the number of future elections one's party may expect to win. Another reason for parties to care
about their electoral margins is that in many legislative settings a party with a large majority of the seats has an easier time implementing policy, both because of supermajority clauses on some policy initiatives, such as constitutional amendments and bills that have been subject to a presidential veto, and because defections are a more serious threat when margins are slim, and every legislator is needed to pass the government's program.

All these issues can be combined by making each party's objective a function of three things: its valuation of the policy outcomes that follow victory and defeat—\( S(C^L,X^L) \) and \( S(C^R,X^L) \), respectively—and the probability of victory which is itself a function of its vote share—\( V^L(C^L,C^R) \). Thus, for party \( L \) we have a function,

\[
F^L(S(C^L,X^L), S(C^R,X^L), V^L(C^L,C^R)),
\]

and similarly for party \( R \). The conventional formulation, which uses the expected utility of the outcome, is a special case of this.

For future use we let \( \beta^L \) denote party \( L \)'s marginal rate of substitution between its social welfare valuation of its own platform and its vote share: \( \beta^L = F^L_3/F^L_1 \), where subscripts as usual denote partial derivatives with respect to the indicated arguments. Thus, \( \beta^L \) measures the power hunger of party \( L \). Party \( R \)'s \( \beta^R \) is defined and interpreted analogously. The \( \beta \)'s are endogenous.

I. Sincere and Strategic Voting

Our model can work with either of two common modeling approaches that specify the rules by which election outcomes are determined, and find the optimal voting behavior under each rule. In the first, the platform of the winner becomes the policy, and the probability of a party winning is an increasing function of its vote share given by formula (7). This can be justified by supposing that, in addition to the known groups of voters who are explicitly modeled above, there are groups of extremist voters: for example, voters for whom only social welfare concerns matter, and who therefore adhere to one party or the other, regardless of self-interest or idiosyncratic preference shocks. The number of these extremist voters at each end of the political spectrum is unknown, and the parties treat it as a random variable. We can then take expectations over the distributions of these random voters; see Alesina and Rosenthal [1995, p. 24]. In the second approach to modeling postelection policy, the outcome is a weighted average of
the platforms of the two parties, say \( pC^L + (1 - p)C^R \), where \( p \) is an increasing function of \( V^L \). This might emerge from extended bargaining between election winners with reciprocal vetos on policy implementation.

If the first approach is taken, sincere voting is also optimal. If the second approach is taken, a weighted average of consumption vectors does not map linearly into any voter's evaluation of social welfare, and sincere voting is not optimal. However, if the election is close in the sense that the probability of each party's winning is near \( \frac{1}{2} \), then sincere voting is close to optimal. The formal proof is in Appendix 1; also see Grossman and Helpman [1996, Section 4]. Since a close election is the only situation where the decision of a single voter in a large population makes any difference, we think it reasonable to adopt this approximation.

In either case, then, Appendix 1 shows that the cut-points are given by

\[
X_i = \frac{1}{\Delta} \left[ \alpha_i(C_i^L - C_i^R) + \frac{1}{2} \sum_k N_k(C_k^R - \hat{C}^R)^2 - \frac{1}{2} \sum_k N_k(C_k^L - \hat{C}^L)^2 \right],
\]

where

\[
\Delta = \frac{1}{2} \left[ \sum_k N_k(C_k^R - \hat{C}^R)^2 - \sum_k N_k(C_k^L - \hat{C}^L)^2 \right. \\
+ \left. \sum_k N_k(C_k - Y_k)^2 - \sum_k N_k(C_k^R - Y_k)^2 \right].
\]

Should parties' platforms converge, matters would become complex. If the parties adopt identical platforms, then everyone will be indifferent between them, the vote split is arbitrary,\(^4\) and we can set \( \nu^L = \nu^R = \frac{1}{2} \). However, if one party were to deviate even slightly, the resulting vote-shares would depend on the direction of the deviation.\(^5\) This will have implications for equilibrium if the two parties are located too close to each other; we will discuss this later.

\(^4\) Formally, if \( C^L = C^R \), then by equation (9) the cut-point \( X_i \) becomes indeterminate \((0/0)\).

\(^5\) Technically we can represent this by holding one party's strategy, say \( C^L \), fixed, and letting the other's, \( C^R \), approach it. The limit of \( X_i \) depends on the direction of the approach; there is a discontinuity in the vote share at the point of identical strategies.
III. TAX AND TRANSFER POLICIES

Now we turn to the parties’ platform choices. We consider the strategy of party L; that of party R follows exactly the same steps. The choice is subject to constraint (1). We confine analysis to interior solutions; corner solutions are a matter of tedious enumeration and are governed by broadly similar principles. We will make brief remarks about corner solutions later.

To write the first-order conditions for party L’s maximization, we introduce some notation. Let $\lambda^L > 0$ be the marginal social value of constraint (1) measured in the units of party L’s evaluation of social welfare. Thus, it tells us how severely the constraint binds. Of course $\lambda^L$ is endogenous, to be determined as a part of the solution.

Next, we define the population proportion weighted average across groups of the densities at the cut-points:

$$\hat{\phi} = \sum_i N_i \phi_i(X_i).$$

The weighted average of the cut-points themselves, with weights proportional to the numbers $N_i \phi_i(X_i)$ located at the cut-points, is denoted by

$$X^* = \frac{\sum_i N_i \phi_i(X_i) X_i}{\sum_i N_i \phi_i(X_i)}.$$  

In each group the voters located at the cut-points are the pivotal ones for whose support both parties compete at the margin. Therefore, $X^*$ can be regarded as the average of the ideology of pivotal voters in the population.

It is important to note that $X^*$ is not the average ideology in the population as a whole; the latter would be

$$\bar{X} = \sum_i N_i \int_0^1 x_i \phi_i(x_i) \, dx_i.$$ 

To clarify the difference, note that for the richer groups, that is, those with high $Y_i$, the cut-point $X_i$ is more likely to be low, because only the most staunchly liberal rich, who are sufficiently willing to sacrifice their own private consumption, will remain indifferent between the leftist and the rightist parties. But the
average ideology of the rich group as a whole is likely to be much farther to the right. The opposite holds for the poor groups; their average ideology will have small $X$, but the pivotal point will be quite far to the right. This point will become clearer when we solve explicitly for the cut-points in Section IV.

Finally, we write

$$
\pi_i = \alpha_i \phi_i(X_i)
$$

and

$$
\hat{\pi} = \sum_i N_i \pi_i.
$$

The $\pi_i$ measure the political "clout" of the various groups, and are exactly analogous to the corresponding parameters introduced and discussed in Dixit and Londregan [1996]. Each $\pi_i$ is the product of two things: $\alpha_i$, which is a measure of selfishness of each member of group $i$, and $\phi_i(X_i)$, which measures the relative density of voters from this group at its cut-point. Then $\hat{\pi}$ is just the average clout in the population, and it is the excess or deficit of any one group’s clout relative to the average that will govern its success in tactical redistributive politics.

The cut-points must be determined endogenously in the equilibrium. Therefore, the magnitudes $\pi_i$, $\hat{\pi}$, $\phi$, and $X^*$ are all also endogenous. Labeling them in this way does not directly contribute to solving for the equilibrium, but it does simplify the algebra and helps give some intuition for the results.

It is easier to express the result in terms of taxes and transfers. Define the total net tax (tax paid minus transfers received) for a member of group $i$ as

$$
T_i = Y_i - C_i,
$$

and its population average

$$
\hat{T} = \sum_i N_i T_i.
$$

(Note that $\hat{T}$ is generally not zero, but positive, because of deadweight losses.)

With all this notation we state and interpret the results; the mathematical details are in Appendix 2. The first-order condition
of party $L$ for group $j$ yields

$$T_j^L - \hat{T}_j^L = \frac{(1 - X_j^L) + (\beta_j^L \phi_j^L)/(\Delta_j) (1 - X_j^*)}{1 + (\beta_j^L \phi_j^L) + \delta j^L} (Y_j - \hat{Y})$$

$$- \frac{\beta_j^L / \Delta}{1 + (\beta_j^L \phi_j^L) + \delta j^L} (\pi_j - \hat{\pi}).$$

A precisely analogous expression will hold for party $R$.

We emphasize that this is not yet a complete solution; it is merely a rewriting of party $L$’s first-order condition. Many magnitudes on the right-hand side are endogenous to the equilibrium, and have not yet been determined. Nevertheless, the expression is very instructive. The endogenous magnitudes $\phi_j, \Delta, X_j^*, \beta_j^L$, and $\lambda_j^L$ (also $\beta_j^R$ and $\lambda_j^R$ in the corresponding expression for party $R$) are the same for all groups $j$. Therefore, the expression yields valid results about the comparative treatment of different groups by each party. We now state and discuss such results.

To interpret the results, we need to sign $\Delta$. We expect party $R$ to generate more variance of consumption than party $L$, and party $L$ to generate more distortion than party $R$, so $\Delta$ should be positive. However, each party may compromise its ideology for the sake of votes, and put forth a platform embodying a degree of inequality very different from what it would most prefer. If one party is significantly more power hungry than the other, so that the parties compromise to very different degrees, then $\Delta > 0$ may not hold. In Appendix 3 we develop precise conditions to ensure $\Delta > 0$.

The first term on the right-hand side of (12) depends on income; this is the ideological component of redistributive taxation. The second term depends on the measure of clout; this is the tactical component. Most importantly, the whole expression looks exactly like a linear formula for income taxation. The coefficient of $Y_j$ on the right-hand side, which can then be thought of as the marginal tax rate, does not depend on the group identification, and therefore applies to the population as a whole. The constant term, which is a sum each person pays or receives, does depend on group identification.\(^6\)

\(^6\) This result, though appealing in view of the frequent arguments in the public policy debate for a flat tax, does depend on the parametric assumptions of the model. For example, if the deadweight loss parameter $\delta_i$ could differ for
Thus, we see that the policy promised by party \( L \) can be implemented as a combination of a general income tax, which serves the ideological purpose, and group-specific transfers, which serve the tactical or pork-barrel purpose. This separability is admittedly a consequence of our choice of functional forms, but it is interesting to note that redistributive policy in reality shows just such a separation. The income tax is understood to perform the equalizing role, and there are separate handouts in cash or kind targeted at specific groups; for example, farm subsidies and senior citizens' health care.

Of course, the two parties' strategies have different marginal tax rates, and also different group-specific transfers. Note that the parties have not abandoned their ideologies \( X^L \) and \( X^R \). They have merely committed themselves to policies that embody some political compromises for the sake of electoral advantage. Public debate often confuses the two aspects.

More insight comes from closer inspection of the marginal income tax rate in this formula. First, consider a case where there are no distortionary losses, so \( \delta = 0 \). Then the marginal tax rate is a weighted average of \((1 - X^L)\) and \((1 - X^*)\), with relative weights of 1 and \((\beta^L \phi/\Delta)\), respectively.

First, we examine the two limits of this average, \((1 - X^L)\) and \((1 - X^*)\), separately. The former is the tax rate that party \( L \) would declare if it followed its own ideological preferences without regard to electoral success. The latter rate corresponds to the average preference of the pivotal voters, as measured by the average of the cut-points \( X^* \). As one would expect, each tax rate is smaller when the corresponding ideology parameter is larger; that is, when the ideology is inclined more toward the right.

The actual tax rate promised by party \( L \) is a compromise, the weight on the pivotal voters' preferred rate being larger when \((\beta^L \phi/\Delta)\) is larger. This makes excellent sense. (i) A large \( \beta^L \) shows that party \( L \) has greater power hunger and therefore is more willing to compromise its ideology in the direction of the crucial pivotal voters' preference. This is the way the British Labour Party has moved in the last year or so. (ii) A larger \( \phi \) shows that a larger proportion of the electorate is at the cut-points. Therefore,

different groups, and the rich had lower \( \delta_i \), then the tax rate on their incomes could be higher. This would create a positive model of tax progressivity, based on political calculation rather than ethical arguments.

there is greater payoff in terms of votes from following their wishes. (iii) A smaller $\Delta$ means that the two parties are relatively close to each other. Therefore, pivotal voters change their votes more readily in response to marginal inducements by either party. This increases the political payoff from following their wishes.

Of course, all the magnitudes in $(\beta^L \phi/\Delta)$ are endogenous. Therefore, these interpretations need some care. For example, $\beta^L$ is the marginal rate of substitution between the left party’s concern for its vote share and its social welfare valuation of its platform. Therefore, the reason for $\beta^L$ being high must ultimately be traced back to an assumption on the slope of the indifference curves in the party’s objective function (8). In the next section we construct an explicit solution for a special case, where all comparative statics statements can be expressed with respect to truly exogenous parameters. Those results support the intuition gained by the above inspection of (12).

If distortions exist, so $\delta > 0$, and if the social budget constraint (1) binds more tightly, that is, $\lambda_L$ is larger, then the denominator of the expression for the marginal tax rate increases. The weights attached to $(1 - X^L)$ and $(1 - X^*)$ sum to less than 1. Considerations of economic efficiency temper the party’s desire to levy redistributive taxation for ideological reasons—pursuit of its own ideology as well as pursuit of votes by catering to the ideology of the pivotal voters.

Turning to the tactical redistribution term, we see that groups with above-average clout pay smaller taxes than the average (or receive transfers larger than the average). The coefficient, which shows the excess transfer per unit of excess clout, is increasing in $(\beta^L/\Delta)$. The reason is the same as for the ideological redistribution term: if the party is more power hungry, or if the voters are more responsive to promises of transfers, then the party is more ready to offer such transfers to the politically powerful groups.

The coefficient decreases as $\delta$ or $\lambda^L$ increases. Thus, the prospect of economic inefficiency tempers tactical redistribution just as it does the ideological kind. Previous electoral models of pork-barrel politics, for example, Cox and McCubbins [1986], Lindbeck and Weibull [1987], and Dixit and Londregan [1995, 1996], all ignored distortionary taxation. Our analysis shows how this realistic consideration modifies those analyses.

One might expect that if the two parties are equally power hungry ($\beta^L = \beta^R$), then the leftist party would want to introduce
more distortion: therefore, that constraint (1) would bind more tightly for it, and that $\lambda^L > \lambda^R$. Then the coefficient of $(\pi_j - \hat{\pi})$ in (12) would be smaller for party $L$ than that for party $R$. In other words, the leftist party’s ideology would prove a handicap in pork-barrel politics. However, that is not necessarily true. It is possible that some particular configuration of parameters yields transfers for pork-barrel reasons which are almost the exact opposite of those for egalitarian reasons. If such is the case, then the leftist party’s overall policy will have less distortion than that of the rightist party. We have not yet been able to find the precise conditions that determine when this will happen.

A. Director’s Law

The disproportionate success of the middle classes at the expense of both the rich and the poor in redistributive politics has often been noted. It was labeled Director’s Law by Stigler [1970]. The empirical basis for this claim depends on how one evaluates noncash transfers, and indirect effects of regulation, public projects, etc. Stigler offers some supporting evidence, but the Economic Report of the President [1992, pp. 134–138] argues that overall redistribution is purely progressive.

Stigler’s analysis of the politics of redistribution [1970, pp. 8–9] is somewhat vague. The result is a progressive income tax at the median voter’s most preferred rate, rather than redistribution of resources from the poor and the wealthy to the middle income classes. He also assumes that voters care only about private consumption, and parties care only about winning.

Our model provides a more general and more precise structure for thinking about this issue. We allow heterogeneity among voters, who then make different trade-offs among personal consumption, social equity, and efficiency. We find that middle-income groups should fare well because they may be expected to possess traits treated favorably by tax formula (12). In addition to their sheer numbers (at least in the advanced economies), the middle classes happen to have two additional features that combine to increase their attractiveness to the political parties.

First, each of the extreme groups, the rich and the poor, gets strongly attached to one party’s redistributive ideology because of their concerns about private consumption. The rich are more attached to the rightist party, because they will receive more private consumption if it prevails. Therefore, their cut-points are well to the left; only the most staunchly liberal rich will vote for
the left party. Similarly, the cut-points for low-income groups are far to the right.

Second, we may expect that because of their personal experiences the rich will tend to place more weight on efficiency concerns in their social welfare functions, while the experience of having less than those around them will leave the poor more concerned with equity. Evidence supporting this view was cited before [McClosky and Zaller 1984, pp. 154–155; Moffitt, Ribar, and Wilhelm 1996, Figure 5]. In other words, the distribution of wealthy voters will have its mass more in the rightist region, and the distribution of the poor will be similarly concentrated to the left. In both cases, the bulk of voters is located away from the cut-point, rightward for the affluent and leftward for the poor. But, by similar reasoning, middle-income groups have both the mass of their distribution and their cut-point in a middle region. Figure 1 shows the relationships between the densities and the cut-points for the three groups.

The combined effect is a low density $\phi_i(X_i)$ at the cut-points for both high- and low-income groups, and a high density for the middle group. Tax formula (12) reflects this in two ways.

First, the pivotal center of politics, $X^*$, is very strongly influenced by the middle class’s most preferred ideology. Therefore, the parties will compromise their platforms toward this ideology, and the general tax rate will be close to the middle class’s most preferred rate. This result is similar to that of median voter models, but the route is different, and the weighting by the densities at the cut-points makes the middle classes even more influential than their sheer numbers in the population would suggest. Our model, in common with the literature, assumes that everyone votes. But if the poor are less likely to vote, then their weight in the formula will be further reduced, and the general taxation will be even less egalitarian.8

The second effect of the middle class’s centrality is even more stark. The clout parameter $\pi_i$ in tax equation (12) is low for the high- and low-income groups, making them net payers, and high for the middle-income groups, making them net beneficiaries, of pork-barrel redistribution. The intuition is that the right party takes the support of the rich for granted, and writes off the poor;

8. The empirical analysis of Bartels [1995] supports the importance of centrality as a determinant of political power. He also notes the effect of participation, namely the lower turnout of the poor and nonpartisan citizens.
the left party takes the poor for granted, and writes off the rich; both parties compete for the support of the middle group.

IV. EQUILIBRIUM

Tax formula (12) proved instructive, but it left many equilibrium magnitudes endogenous. We now construct the complete
equilibrium for a special case of our model. The general case follows similar steps, but the algebra is more complex, and the solution is not in closed form but a set of simultaneous equations. Fortunately, the special case suffices to convey several insights.

In this special case, we ignore distortions, setting $\delta = 0$. We assume that the parties’ objective functions are linear in their three arguments, for example:

$$F^L = K + S(C^L, X^L) + \gamma L S(C^R, X^L) + \beta L V^L(C^L, C^R),$$

where $K$, $\gamma L$, and $\beta L$ are constants, with $\beta L$ positive. This makes power hunger an exogenous parameter. We also assume that the two parties are equally power hungry, so that $\beta L = \beta R$; we write $\beta$ for their common value. Of course, the two must have different ideologies, that is, $X^L \neq X^R$; this is an essential aspect of ideological politics.

Finally, we assume that each group in the electorate has uniform density over an interval of the ideological spectrum. Thus, for some $a_i, b_i$ satisfying $0 \leq a_i < b_i \leq 1$, we have

$$\phi_i(x) = \begin{cases} 
\frac{1}{b_i - a_i} & \text{for } a_i \leq x \leq b_i \\
0 & \text{otherwise}.
\end{cases}$$

With these assumptions, the equilibrium is as follows; for details of the algebra see Appendix 4.

The measure of the difference between the parties is

$$(13) \quad \Delta = (X^R - X^L) \operatorname{var} (Y) - \beta \hat{\phi},$$

where

$$\operatorname{var} (Y) = \sum_i N_i (Y_i - \hat{Y})^2$$

is the population variance of productivity.

All the magnitudes on the right-hand side of (13) are exogenous, and it is perfectly possible that they yield a negative value of $\Delta$. We prove in Appendix 3 that in any equilibrium where the parties’ strategies are distinct, $\Delta$ must be positive. But there cannot be an equilibrium where the parties pursue identical pure strategies, because starting from such a situation either party can choose a small deviation to achieve a discontinuous gain in its vote share. Therefore, for those configurations of parameters that yield a negative expression in (13), there cannot be an equilibrium in pure strategies at all. This happens if $(X^R - X^L)$ is sufficiently
small. Therefore, the problem is precisely analogous to the nonexistence of equilibrium in a Hotelling-type model of price competition when two sellers are located close to each other; see D'Aspremont, Gabszewicz, and Thisse [1979].

Let us examine the equilibrium when it does exist; that is, when the solution for \( \Delta \) in (13) is positive. The first term on the right-hand side of this expression shows the force of ideological politics. If the parties' ideologies are far apart, or if the variance of gross earnings in the population is large, then the parties will differ substantially in the egalitarianism of the outcome of their tax and transfer policies, and \( \Delta \) will be large. The second term shows the tactical or pork-barrel aspect of redistributive politics. If the parties are more power hungry, or if the voters' ideologies are more concentrated so more of them move in response to private consumption inducements, then both parties will chase after their votes and \( \Delta \) will be small. (We have assumed uniform densities; in a more general context only the densities at the endogenously determined cut-points will matter.)

The cut-points of the various groups are given by

\[
X_i = \frac{1}{2} (X_L + X_R) - \alpha_i (Y_i - \bar{Y}) / \text{var}(Y).
\]

Here the first term shows the ideological aspect of redistribution. If the \( \alpha_i \) were zero, so that voters cared only about distributive ideologies and not at all about their own personal consumption levels, then they would support the party whose ideology parameter \( X \) was closer to their own. Therefore, all groups' cut-points would fall halfway between \( X_L \) and \( X_R \). The second term on the right-hand side of (14) shows how the voters' selfish motives alter this. Groups that are richer than average have their cut-points to the left of this midpoint: only a sufficiently strongly egalitarian rich voter would remain indifferent between the two parties, despite the fact that the left party would levy a higher tax on him/her. Similarly, only a sufficiently strongly rightist poor voter would be found at the cut-point of his or her group. This effect is stronger for group \( i \) when \( \alpha_i \) is larger; that is, when the voters in this group value their own consumption more relative to social welfare. It is weaker when the variance of gross earnings is higher, because then inequality creates a greater social loss in the calculation of each voter with his or her given \( X \).
Formula (14) may yield a negative value of $X_i$ for some group $i$ with a sufficiently large $Y_i$; then no members of that group would vote for party $L$. Similarly, the formula may yield cut-points above 1 for some very poor groups; then all the members of such groups would vote for party $L$. Of course, in such cases we would need to replace the $X_i$ generated by (14) by 0 or 1 as appropriate, and rework the entire equilibrium calculation to verify that the corner solution was fully internally consistent. This is a tedious task, and the general intuition about the relationship between cut-points and earnings is adequately conveyed by the interior solution of (14). Therefore, we have ignored corner solutions.

The density-weighted average of the cut-points of the pivotal voters is calculated using definition (11) as

$$X^* = \frac{1}{2} (X^L + X^R) - (\hat{\phi})^{-1} \text{cov} (Y, \pi) / \text{var} (Y),$$

where

$$\text{cov} (Y, \pi) = \sum_i N_i (Y_i - \hat{Y})(\pi_i - \hat{\pi})$$

is the covariance between earnings and clout. Thus, if the richer groups have greater clout on average, then the average cut-point of the pivotal voters is to the left of the midpoint between the parties’ positions. The reason follows from that for the location of the cut-points of the individual groups. We saw that a rich group will have its cut-points to the left, and the strength of this is proportional to the group’s selfishness parameter $\alpha_i$. The contribution this makes to the average across groups further depends on the density $\phi_i$. Thus, the overall effect depends on the product $\alpha_i \phi_i$, which is just the clout parameter $\pi_i$. Similarly, a poor group $j$ will have its cut-point to the right, and its contribution to the average cut-point will be proportional to its clout. Adding across groups, the average cut-point is to the left of the midpoint between the parties if the rich groups have higher clout on average. However, as we pointed out above, we expect the middle classes to have most political power, so the covariance should be small, and $X^*$ should be close to the midpoint.

Finally, we can complete the analysis by substituting these results into the formula (12) for the tax and transfer policy. In
particular, the marginal tax rate chosen by party $L$ can be written as
\[
(1 - X^L) - \beta \frac{\hat{\phi}}{2 \operatorname{var}(\mathbf{Y})} - \frac{\beta}{X^R - X^L} \frac{\operatorname{cov}(\mathbf{Y}, \pi)}{\operatorname{var}(\mathbf{Y})^2}.
\]

The first term is the tax rate the party would have chosen on the basis of its ideology alone if it did not care for electoral success. The second term shows the electoral effects of pork-barrel politics. A greater power hunger, and a greater density of pivotal responsive voters, leads the party to choose a lower tax rate, and a larger variance of pretax incomes causes it to choose a higher tax rate. However, this aspect on its own would always keep the tax rate below the party ideological ideal. The third term shows the electoral effects of the voters' distributive ideologies. If the poorer groups are politically more powerful (negative covariance between $\mathbf{Y}$ and $\pi$), that causes the party to choose a higher tax rate.

Thus, the closed-form equilibrium solution for this case shows how the intuition generated by first-order condition (12) carries through to the general equilibrium level.

V. Concluding Comments

We have constructed a model of electoral competition where the voters and the parties care, not just about their own consumption and votes, but also about the distribution of income. We consider the joint effects of social and selfish motives on political outcomes. In other words, we offer a joint analysis of ideological and pork-barrel politics.

We have shown how parties compromise their own ideology so as to get more votes, adjusting both the ideological and pork-barrel components of policy to cater to the pivotal voters—those located at the cut-points in the various groups, weighted by the numbers at these cut-points. How responsive a party's platform is to the desires of the pivotal voters depends directly on the power hunger of the party, and inversely on the degree of polarization between the two parties. As in earlier models of pork-barrel politics in which the parties' ideological positions had no fiscal implications and were fixed exogenously, the parties offer pork-barrel transfers to members of various identifiable groups, granting more favorable treatment to groups with higher concentrations of pivotal voters, and those whose cut-points shift more
readily in response to the promises of transfers. Parties also take advantage of the opportunity for ideological compromise, moving their platforms away from their ideologies, in the direction of a point that can be regarded as the *pivotal* center of ideology in the electorate. This point is not the average of the ideology in the population as a whole, but the average of the ideologies of the pivotal voters—those located at the cut-points in the various groups, weighted by the numbers at these cut-points.

The overall strategy of each party can be implemented by levying an income tax at a marginal rate that is common for all groups, and transfers (positive or negative) whose size is group-specific. Actual distributive policies often exhibit just such separation.

Another feature of reality is the success of the middle classes in redistributive politics. Our model explains it in a simple and natural way: the cut-points of middle classes lie close to their densest concentration in the ideology spectrum; the cut-points for the rich as well as the poor, are far from their respective concentrations of density. Therefore, the parties’ platforms reflect middle-class ideology, and their transfer policies favor these classes at the expense of both the rich and the poor.

We had to make some restrictive assumptions and choose some special functional forms to make the analysis tractable. But the most important of these, namely the quadratic loss functions used in the rightist and leftist social welfare measures (2) and (3), are common in political economy models. Also, the results are appealing enough to create some confidence in the setup. Therefore, we hope that the model will serve as a starting-point for further work. For example, other (political or social) dimensions of ideology can be introduced, and trade-offs between these aspects and economic redistributive ideology can be studied. Some restrictions on instruments, where taxes or transfers are required to be common across some groups, could be introduced. Finally, the economic structure of the model, most particularly the structural link between the taxes and the distortions, can be spelled out in greater detail.

**APPENDIX 1: VOTER CHOICE AND CUT-POINTS**

First, consider the case where the winning party’s policy is implemented. Suppose that the probability *p* of the left party’s victory is an increasing function of its vote share $V^L(C^L,C^R)$. The
expected welfare $EW(X,i)$ of a voter of type $X$ in group $i$ is

$$EW(X,i) = p[\alpha_iC^L_i + S(C^L_i,X)] + (1 - p)[\alpha_iC^R_i + S(C^L_i,X)].$$

If this person votes for party $L$, that will increase $p$, albeit only infinitesimally. That is in the interests of this voter if $EW(X,i)$ increases as $p$ increases, which is equivalent to

$$\alpha_iC^L_i + S(C^L_i,X) > \alpha_iC^R_i + S(C^L_i,X).$$

Thus, sincere voting is optimal in this case.

Indifference defines the cut-point $X_i$. Writing out social welfare expressions using (5), we have

$$(16) \quad \alpha_iC^L_i - \frac{1}{2}X_i \sum_k N_k(C^L_k - Y_k)^2 - \frac{1}{2} \sum_k (1 - X_i) N_k(C^L_k - \hat{C}^L_k)^2
= \alpha_iC^R_i - \frac{1}{2}X_i \sum_k N_k(C^R_k - Y_k)^2 - \frac{1}{2} \sum_k (1 - X_i) N_k(C^R_k - \hat{C}^R_k)^2.$$ 

Solving for $X_i$ gives us formula (9) of the text, along with definition (10) of $\Delta$.

Next consider the case where the actual policy implemented is a compromise between the two platforms:

$$C = pC^L + (1 - p)C^R,$$

where $p$ is as above. Now

$$EW(X,i) = \alpha_i[pC^L_i + (1 - p)C^R_i] + S(pC^L + (1 - p)C^R_i,X).$$

Routine algebra yields

$$\frac{\partial EW(X,i)}{\partial p} = \alpha_i(C^L_i - C^R_i) - (1 - X) \sum_i N_i[p(C^L_i - \hat{C}^L_i)
+ (1 - p) (C^R_i - \hat{C}^R_i)][(C^L_i - \hat{C}^L_i) - (C^R_i - \hat{C}^R_i)]
- X \sum_i N_i[p(C^L_i - Y_i) + (1 - p)(C^R_i - Y_i)]
\times [(C^L_i - Y_i) - (C^R_i - Y_i)].$$

When the election is close ($p \approx 1/2$), this reduces to

$$\frac{\partial EW(X,i)}{\partial p} = \alpha_i(C^L_i - C^R_i) - \frac{1}{2}X \sum_i N_i[(C^L_i - \hat{C}^L_i)^2 - (C^R_i - \hat{C}^R_i)^2]
- \frac{1}{2} (1 - X) \sum_i N_i[(C^L_i - Y_i)^2 - (C^R_i - Y_i)^2].$$
The voter should vote for party L if this is positive, and for party R if this is negative. Indifference yields the same formula for the cut-points $X_i$ as in the winner-take-all case. Grossman and Helpman [1996] also find that sincere voting is approximately optimal when the two parties’ vote shares are almost equal.

APPENDIX 2: THE PARTIES’ FIRST-ORDER CONDITIONS

Party L chooses $C^L$ to maximize (8) subject to the constraint (1). The Lagrangian for this problem is

$$\mathcal{L}^L = F^L(S(C^L, X^L), S(C^R, X^L), V^L(C^L, C^R)) - \tilde{\lambda}^L H(C^L),$$

where $\tilde{\lambda}^L \geq 0$ is the Lagrange multiplier, to be determined as a part of the solution.

The first-order conditions are

$$F_1^L \frac{\partial S(C^L, X^L)}{\partial C^L_j} + F_3^L \frac{\partial V^L(C^L, C^R)}{\partial C^L_j} - \tilde{\lambda}^L \frac{\partial H(C^L)}{\partial C^L_j} = 0,$$

for all groups $j$. Here the arguments of the partial derivatives $F_1^L$ and $F_3^L$ of $F^L$ are omitted for the sake of brevity, and understood to be the same as those in (8) and in the statement of the Lagrangian. The first-order conditions can be written as

$$\frac{\partial S(C^L, X^L)}{\partial C^L_j} + \beta^L \frac{\partial V^L(C^L, C^R)}{\partial C^L_j} - \lambda^L \frac{\partial H(C^L)}{\partial C^L_j} = 0,$$

where $\beta^L = F_3^L/F_1^L$ measures the power-hunger of the left party in equilibrium, as explained in the text, and $\lambda^L = \tilde{\lambda}^L/F_1^L$ is the Lagrange multiplier measured taking the left party’s social welfare evaluation of its platform as the numeraire.

To simplify and interpret the first-order conditions, we begin by finding how the various components respond to changes in the consumption quantities.

The case of the rightist social welfare function is the simplest; for any group $j$ we have

$$\frac{\partial S^R(C)}{\partial C_j} = -N_j(C_j - Y_j).$$

For the leftist welfare function we must remember that the
average $\hat{C}$ is affected by all the $C_j$. Writing the function as

$$S^L(C) = -\frac{1}{2}\left[\sum_i N_i(C_i)^2 - \left(\sum_i N_iC_i\right)^2\right],$$

we have

$$\frac{\partial S^L(C)}{\partial C_j} = -N_jC_j + \left(\sum_i N_iC_i\right)N_j = -N_j(C_j - \hat{C}).$$

Finally, we need to know how the cut-points respond. Multiplying (9) for group $i$ by $\Delta$ and differentiating with respect to $C^L_j$, and using the above expressions for the derivatives of the social welfare functions, we have

$$\Delta \frac{\partial X_i}{\partial C^L_j} + X_i[N_j(C^L_j - Y_j) - N_j(C^L_j - \hat{C}^L)] = \alpha_i \delta_{ij} - N_j(C^L_j - \hat{C}^L),$$

where

$$\delta_{ij} = \begin{cases} \alpha_j & \text{if } i = j \\ 0 & \text{if } i \neq j. \end{cases}$$

Using all these components, the first-order conditions for the left party can be written as

$$\beta^L \sum_i N_i \phi_i(X_i) \frac{1}{\Delta} [\alpha_i \delta_{ij} - X_iN_j(C^L_j - Y_j) - (1 - X_j)N_j(C^L_j - \hat{C}^L)]$$

$$- X^L N_j(C^L_j - Y_j) - (1 - X^L)N_j(C^L_j - \hat{C}^L)$$

$$- \lambda^L [N_j + \delta N_j(C^L_j - Y_j)] = 0.$$
Subtracting this from each first-order condition and regrouping terms, we get

\[
1 + \left( \frac{\beta L \lambda}{\Delta} \right) + \delta \lambda L (C_L - \hat{C}_L) = \left[ X^L + \left( \frac{\beta L \lambda}{\Delta} \right) X^* - \delta \lambda L \right] (Y_j - \hat{Y}) \\
+ \left( \frac{\beta L}{\Delta} \right) (\pi_j - \hat{\pi}).
\]

This shows how members of different groups fare in their consumption relative to the average. Subtracting this from \((Y_j - \hat{Y}_L)\) gives result (12) in the text.

**APPENDIX 3: PROOF OF \(\Delta > 0\)**

The proof is a “revealed preference” argument. The parties face the same constraint (1). Therefore, each party's strategy would have been feasible for the other party. Then the value of each party's objective function must be at least as high using its own chosen strategy as it would be trying to mimic the other party's strategy. Therefore,

\[
F(L(S(C_L, X_L), S(C_R, X_L), V_L(C_L, C_R)) > F(L(S(C_R, X_R), S(C_R, X_L), 1/2)
\]

and

\[
F(R(S(C_R, X_R), S(C_L, X_R), V_R(C_L, C_R)) > F(R(S(C_L, X_L), S(C_L, X_R), 1/2).
\]

Now we assume that \(F_L\) and \(F_R\) are concave in their arguments. Weak concavity will do; this covers the special case of Section IV where the functions are taken to be linear. For the left party, concavity implies that

\[
F(L(S(C_L, X_L), S(C_R, X_L), V_L(C_L, C_R)) = F(L(S(C_R, X_R), S(C_R, X_L), 1/2)
\]

\[
+ F_1[L(S(C_L, X_L) - S(C_R, X_L)] + F_3[L(V_L(C_L, C_R) - 1/2]
\]

where \(F_1\) and \(F_3\) are evaluated at \((S(C_R, X_L), S(C_R, X_L), 1/2).\) Combining this with the revealed preference inequality for the left party yields

\[
F_1[L(S(C_L, X_L) - S(C_R, X_L)] + F_3[L(V_L(C_L, C_R) - 1/2] > 0,
\]

or, dividing by \(F_3^L,\)

\[
\frac{F_1^L}{F_3^L} [S(C_L, X_L) - S(C_R, X_L)] + [V_L(C_L, C_R) - 1/2] > 0.
\]
Now we write a similar inequality for party $R$, and add the two. Using $V^L + V^R = 1$, this yields

$$
\frac{F^L}{F^L_3} [S(C^L, X^L) - S(C^R, X^L)] + \frac{F^R}{F^R_3} [S(C^R, X^R) - S(C^L, X^R)] > 0.
$$

The marginal rates of substitution multiplying the terms on the left-hand side are reciprocals of $\beta^L$ and $\beta^R$. However, they are evaluated at points of equal vote shares rather than at the equilibrium point. If these magnitudes are almost equal, that is, if the two parties are uniformly almost equally power hungry, then we can conclude that

$$
\Delta = [S(C^L, X^L) - S(C^R, X^L)] + [S(C^R, X^R) - S(C^L, X^R)] > 0.
$$

The condition of uniformly almost equal power-hunger is sufficient, but not necessary. Also, it is met exactly in the special case examined in Section IV, where the two parties have constant and equal $\beta$'s.

**APPENDIX 4: EQUILIBRIUM**

Here we detail the construction of the equilibrium for the special case of Section IV.

We are assuming that $\delta = 0$, so $\hat{C}^L = \hat{C}^R = \hat{Y}$. Then party $L$’s first-order condition, (19) in Appendix 1, can be written as

$$
C^L_j - \hat{Y} = \rho^L(Y_j - \hat{Y}) + \gamma(\pi_j - \hat{\pi}),
$$

where

$$
\rho^L = \frac{X^L + (\beta \hat{\phi}/\Delta)X^*}{1 + (\beta \hat{\phi}/\Delta)},
$$

and

$$
\gamma = \frac{\beta/\Delta}{1 + (\beta \hat{\phi}/\Delta)}.
$$

Similarly

$$
C^R_j - \hat{Y} = \rho^R(Y_j - \hat{Y}) + \gamma(\pi_j - \hat{\pi}).
$$
Substituting in the definition (10) yields

\[ 2\Delta = \sum_k N_k[\rho^R(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 \]

\[ - \sum_k N_k[\rho^L(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 \]

\[ - \sum_k N_k[-(1 - \rho^R)(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 \]

\[ + \sum_k N_k[-(1 - \rho^L)(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 \]

\[ = [(\rho^R)^2 - (\rho^L)^2 - (1 - \rho^R)^2 + (1 - \rho^L)^2]\var(Y) \]

\[ + 2\gamma[\rho^R - \rho^L + (1 - \rho^R) - (1 - \rho^L)]\cov(Y,\pi) \]

\[ = 2(\rho^R - \rho^L)\var(Y). \]

Also,

\[ \rho^R - \rho^L = \frac{X^R - X^L}{1 + (\hat{\beta}\hat{\phi}/\Delta)}. \]

Therefore,

\[ \Delta[1 + (\hat{\beta}\hat{\phi}/\Delta)] = (X^R - X^L)\var(Y), \]

or

\[ \Delta = (X^R - X^L)\var(Y) - \beta\hat{\phi}. \]

All the magnitudes on the right-hand side are exogenous. Therefore, this is a genuine solution for the equilibrium value of \( \Delta \).

We will also need to simplify the variance terms that appear on the right-hand side of definition (9) of group cut-points; namely,

\[ \sum_k N_k(C_k^R - \hat{C}^R)^2 - \sum_k N_k(C_k^L - \hat{C}^L)^2. \]

This equals

\[ \sum_k N_k[\rho^R(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 - \sum_k N_k[\rho^L(Y_k - \hat{Y}) + \gamma(\pi_k - \hat{\pi})]^2 \]

\[ = [(\rho^R)^2 - (\rho^L)^2]\var(Y) + 2\gamma(\rho^R - \rho^L)\cov(Y,\pi) \]

\[ = (\rho^R - \rho^L)[(\rho^R + \rho^L)\var(Y) + 2\gamma\cov(Y,\pi)]. \]
Next we note that

$$C^R_j - C^L_j = (\rho^R - \rho^L)(Y_j - \hat{Y}).$$

Substituting in the definition (9) of the groups’ cut-points, we have

$$\Delta X_j = (\rho^R - \rho^L)[ -\alpha_j(Y_j - \hat{Y}) + \frac{1}{2}(\rho^R + \rho^L) \text{var}(Y)$$

$$+ \gamma \text{cov}(Y, \pi)].$$

Multiplying by $N_j \phi_j$ and adding over $j$ yields

$$\Delta \hat{\phi}X^* = (\rho^R - \rho^L)(\frac{1}{2} \hat{\phi}(\rho^R + \rho^L) \text{var}(Y) - (1 - \gamma \hat{\phi}) \text{cov}(Y, \pi)).$$

Using

$$\rho^R - \rho^L = \Delta/\text{var}(Y)$$

and

$$\frac{1}{2} \frac{1}{\text{var}(Y)} \left( \frac{X^L + X^R}{1 + \beta \hat{\phi}/\Delta} \right),$$

this simplifies to

$$X^* = \frac{1}{2} (X^L + X^R) - (\hat{\phi})^{-1} \frac{\text{cov}(Y, \pi)}{\text{var}(Y)}.$$

Finally, multiplying (20) by $\hat{\phi}$ and subtracting (21) to get

$$\Delta \hat{\phi}(X_j - X^*) = (\rho^R - \rho^L)[ -\alpha_j(Y_j - \hat{Y})\hat{\phi} + \text{cov}(Y, \pi)],$$

which simplifies to a solution for the group cutpoints:

$$X_i = \frac{1}{2} (X^L + X^R) - \alpha_i(Y_i - \hat{Y})/\text{var}(Y).$$

This is equation (14) in the text, which leads to the expression (15) for $X^*$, the pivotal center of ideology in the electorate. That completes the solution.

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REFERENCES


