Externality

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Externality has been, and is, central to the neo-classical critique of market organisation. In its various forms—external economies and diseconomies, divergencies between marginal social and marginal private cost or product, spillover and neighbourhood effects, collective or public goods—externality dominates theoretical welfare economics, and, in one sense, the theory of economic policy generally. Despite this importance and emphasis, rigorous definitions of the concept itself are not readily available in the literature. As Scitovosky has noted, " definitions of external economies are few and unsatisfactory ".[1] The following seems typical:

> External effects exist in consumption whenever the shape or position of a man's indifference curve depends on the consumption of other men. [External effects] are present whenever a firm's production function depends in some way on the amounts of the inputs or outputs of another firm.[2]

It seems clear that operational and usable definitions are required.

In this paper, we propose to clarify the notion of externality by defining it rigorously and precisely. When this is done, several important, and often overlooked, conceptual distinctions follow more or less automatically. Specifically, we shall distinguish marginal and inframarginal externalities, potentially relevant and irrelevant externalities, and Pareto-relevant and Pareto-irrelevant externalities. These distinctions are formally developed in Section I. As we shall demonstrate, the term, " externality ", as generally used by economists, corresponds only to our definition of Pareto-relevant externality. There follows, in Section II, an illustration of the basic points described in terms of a simple descriptive example. In Section III, some of the implications of our approach are discussed.

It is useful to limit the scope of the analysis at the outset. Much of the discussion in the literature has been concerned with the distinction between *technological* and *pecuniary* external effects. We do not propose to enter this discussion since it is not relevant for our purposes. We note only that, if desired, the whole analysis can be taken to apply only to technological externalities. Secondly, we shall find no cause for discussing production and consumption externalities separately. Essentially the same analysis applies in either case. In what follows, " firms " may be substituted for " individuals " and " production functions "

[1] Tibor Scitovosky, " Two Concepts of External Economies ", *Journal of Political Economy*, vol. LXII (1954), p. 143.

[2] J. de V. Graaf, *Theoretical Welfare Economics*, Cambridge, 1957, p. 43 and p. 18.

for " utility functions " without modifying the central conclusions. For expositional simplicity only, we limit the explicit discussion to consumption externalities.

I

We define an external effect, *an externality*, to be present when,

(1) $u^A = u^A (X_1, X_2, \ldots, X_m, Y_1)$.

This states that the utility of an individual, A, is dependent upon the " activities ", $(X_1, X_2, \ldots, X_m)$, that are exclusively under his own control or authority, but also upon another single activity, Y_1, which is, by definition, under the control of a second individual, B, who is presumed to be a member of the same social group. We define an *activity* here as any distinguishable human action that may be measured, such as eating bread, drinking milk, spewing smoke into the air, dumping litter on the highways, giving to the poor, etc. Note that A's utility may, and will in the normal case, depend on other activities of B in addition to Y_1, and also upon the activities of other parties. That is, A's utility function may, in more general terms, include such variables as $(Y_2, Y_3, \ldots, Y_m; Z_1, Z_2, \ldots, Z_m)$. For analytical simplicity, however, we shall confine our attention to the effects of one particular activity, Y_1, as it affects the utility of A.

We assume that A will behave so as to maximise utility in the ordinary way, subject to the externally determined values for Y_1, and that he will modify the values for the X's, as Y_1 changes, so as to maintain a state of " equilibrium ".

A marginal externality exists when,

(2) $u^A_{Y_1} \neq 0$.

Here, small u's are employed to represent the " partial derivatives " of the utility function of the individual designated by the super-script with respect to the variables designated by the subscript. Hence, $u^A_{Y_1} = \partial u^A / \partial Y_1$, assuming that the variation in Y_1 is evaluated with respect to a set of " equilibrium " values for the X's, adjusted to the given value for Y_1.

An infra-marginal externality holds at those points where,

(3) $u^A_{Y_1} = 0$,

and (1) holds.

These classifications can be broken down into economies and diseconomies: a marginal external economy existing when,

(2A) $u^A_{Y_1} > 0$,

that is, a small change in the activity undertaken by B will change the utility of A in the same direction; a marginal external diseconomy existing when,

(2B) $u^A_{Y_1} < 0$.

An infra-marginal external economy exists when for any given set of values for $(X_1, X_2, \ldots, X_m)$, say, $(C_1, C_2, \ldots, C_m)$,

(3A) $u^A_{Y_1} = 0$, and $\int_0^{Y_1} u^A_{Y_1}\, d_{Y_1} > 0$.

This condition states that, while incremental changes in the extent of B's activity, Y_1, have no affect on A's utility, the total effect of B's action has increased A's utility. An infra-marginal diseconomy exists when (1) holds, and, for any given set of values for $(X_1, X_2, \ldots, X_m)$, say, $(C_1, C_2, \ldots, C_m)$, then,

(3B) $u^A_{Y_1} = 0$, and $\int_0^{Y_1} u^A_{Y_1}\, d_{Y_1} < 0$.

Thus, small changes in B's activity do not change A's level of satisfaction, but the total effect of B's undertaking the activity in question is harmful to A.

We are able to classify the effects of B's action, or potential action, on A's utility by evaluating the "partial derivative" of A's utility function with respect to Y_1 over all possible values for Y_1. In order to introduce the further distinctions between *relevant* and *irrelevant* externalities, however, it is necessary to go beyond consideration of A's utility function. Whether or not a relevant externality exists depends upon the extent to which the activity involving the externality is carried out by the person empowered to take action, to make decisions. Since we wish to consider a single externality in isolation, we shall assume that B's utility function includes only variables (activities) that are within his control, including Y_1. Hence, B's utility function takes the form,

(4) $u^B = u^B(Y_1, Y_2, \ldots, Y_m)$.

Necessary conditions for utility maximisation by B are,

(5) $u^B_{Y_1}/u^B_{Y_j} = f^B_{Y_1}/f^B_{Y_j}$,

where Y_j is used to designate the activity of B in consuming or utilising some numeraire commodity or service which is, by hypothesis, available on equal terms to A. The right-hand term represents the marginal rate of substitution in "production" or "exchange" confronted by B, the party taking action on Y_1, his production function being defined as,

(6) $f^B = f^B(Y_1, Y_2, \ldots, Y_m)$,

where inputs are included as activities along with outputs. In other words, the right-hand term represents the marginal cost of the activity, Y_1, to B. The equilibrium values for the Y_i's will be designated as $\overline{Y}_i$'s.

An externality is defined as *potentially relevant* when the activity, to the extent that it is actually performed, generates *any* desire on the part of the externally benefited (damaged) party (A) to modify the behaviour of the party empowered to take action (B) through trade,

persuasion, compromise, agreement, convention, collective action, etc. An externality which, to the extent that it is performed, exerts no such influence is defined as *irrelevant*. Note that, so long as (1) holds, an externality remains; utility functions remain interdependent.

A potentially relevant marginal externality exists when,

$$(7)\quad u^A_{Y_1}\Big|_{Y_1=\overline{Y}_1} \neq 0.$$

This is a potentially relevant marginal external economy when (7) is greater than zero, a diseconomy when (7) is less than zero. In either case, A is motivated, by B's performance of the activity, to make some effort to modify this performance, to increase the resources devoted to the activity when (7) is positive, to decrease the quantity of resources devoted to the activity when (7) is negative.

Infra-marginal externalities are, by definition, irrelevant for small changes in the scope of B's activity, Y_1. However, when large or discrete changes are considered, A is motivated to change B's behaviour with respect to Y_1 in all cases *except* that for which,

$$(8)\quad u^A_{Y_1}\Big|_{Y_1=\overline{Y}_1} = 0, \text{ and}$$
$$u^A\ (C_1,\ C_2,\ \ldots,\ C_m,\ \overline{Y}_1) \geqq u^A\ (C_1,\ C_2,\ \ldots,\ C_m,\ Y_1), \text{ for all } Y_1 \neq \overline{Y}_1.$$

When (8) holds, A has achieved an absolute maximum of utility with respect to changes over Y_1, given any set of values for the X's. In more prosaic terms, A is satiated with respect to Y_1.[1] In all other cases, where infra-marginal external economies or diseconomies exist, A will have some desire to modify B's performance; the externality is potentially relevant. Whether or not this motivation will lead A to seek an expansion or contraction in the extent of B's performance of the activity will depend on the location of the infra-marginal region relative to the absolute maximum for any given values of the X's.[2]

Pareto relevance and irrelevance may now be introduced. The existence of a simple desire to modify the behaviour of another, defined as potential relevance, need not imply the ability to implement this desire. An externality is defined to be Pareto-relevant when the extent of the activity may be modified in such a way that the externally affected party, A, can be made better off without the acting party, B, being made worse off. That is to say, " gains from trade " characterise the Pareto-relevant externality, trade that takes the form of some change in the activity of B as his part of the bargain.

[1] Note that, $u^A_{Y_1}\Big|_{Y_1=\overline{Y}_1} = 0$, is a necessary, but not a sufficient, condition for irrelevance.

[2] In this analysis of the relevance of externalities, we have assumed that B will act in such a manner as to maximise his own utility subject to the constraints within which he must operate. If, for any reason, B does not attain the equilibrium position defined in (5) above, the classification of his activity for A may, of course, be modified. A potentially relevant externality may become irrelevant and *vice versa*.

A marginal externality is Pareto-relevant when[1]

(9) $(-)\ u^A_{Y_1}/u^A_{X_j} > [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]_{Y_1=\bar{Y}_1}$ and when $u^A_{Y_1}/u^A_{X_j} < 0$, and $u^A_{Y_1}/u^A_{X_j} > (-)\ [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]_{Y_1=\bar{Y}_1}$ when $u^A_{Y_1}/u^A_{X_j} > 0$.

In (9), X_j and Y_j are used to designate, respectively, the activities of A and B in consuming or in utilising some numeraire commodity or service that, by hypothesis, is available on identical terms to each of them. As is indicated by the transposition of signs in (9), the conditions for Pareto relevance differ as between external diseconomies and economies. This is because the " direction " of change desired by A on the part of B is different in the two cases. In stating the conditions for Pareto relevance under ordinary two-person trade, this point is of no significance since trade in one good flows only in one direction. Hence, absolute values can be used.

The condition, (9), states that A's marginal rate of substitution between the activity, Y_1, and the numeraire activity must be greater than the " net " marginal rate of substitution between the activity and the numeraire activity for B. Otherwise, " gains from trade " would not exist between A and B.

Note, however, that when B has achieved utility-maximising equilibrium,

(10) $u^B_{Y_1}/u^B_{Y_j} = f^B_{Y_1}/f^B_{Y_j}$.

That is to say, the marginal rate of substitution in consumption or utilisation is equated to the marginal rate of substitution in production or exchange, i.e., to marginal cost. When (10) holds, the terms in the brackets in (9) mutually cancel. Thus, potentially relevant marginal externalities are also Pareto-relevant when B is in utility-maximising equilibrium. Some trade is possible.

Pareto equilibrium is defined to be present when,

(11) $(-)u^A_{Y_1}/u^A_{X_j} = [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]$, and when $u^A_{Y_1}/u^A_{X_j} < 0$, and $u^A_{Y_1}/u^A_{X_j} = (-)[u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]$ when $u^A_{Y_1}/u^A_{X_j} > 0$.

Condition (11) demonstrates that marginal externalities may continue to exist, even in Pareto equilibrium, as here defined. This point may be shown by reference to the special case in which the activity in question may be undertaken at zero costs. Here Pareto equilibrium is attained when the marginal rates of substitution in consumption or utilisation for the two persons are precisely offsetting, that is, where their interests are strictly opposed, and *not* where the left-hand term vanishes.

What vanishes in Pareto equilibrium are the Pareto-relevant externalities. It seems clear that, normally, economists have been referring only to what we have here called Pareto-relevant externalities when

[1] We are indebted to Mr. M. McManus of the University of Birmingham for pointing out to us an error in an earlier formulation of this and the following similar conditions.

they have, implicitly or explicitly, stated that external effects are not present when a position on the Pareto optimality surface is attained.[1]

For completeness, we must also consider those potentially relevant infra-marginal externalities. Refer to the discussion of these as summarised in (8) above. The question is now to determine whether or not, A, the externally affected party, can reach some mutually satisfactory agreement with B, the acting party, that will involve some discrete (non-marginal) change in the scope of the activity, Y_1. If, over some range, any range, of the activity, which we shall designate by $\triangle Y_1$, the rate of substitution between Y_1 and X_j for A exceeds the " net " rate of substitution for B, the externality is Pareto-relevant. The associated changes in the utilisation of the numeraire commodity must be equal for the two parties. Thus, for external economies, we have

$$(12)\quad \frac{\triangle u^A}{\triangle Y_1}\Big/\frac{\triangle u^A}{\triangle X_j} > (-)\left[\frac{\triangle u^B}{\triangle Y_1}\Big/\frac{\triangle u^B}{\triangle Y_j} - \frac{\triangle f^B}{\triangle Y_1}\Big/\frac{\triangle f^B}{\triangle Y_j}\right]_{Y_1=\overline{Y}_1},$$

and the same with the sign in parenthesis transposed for external diseconomies. The difference to be noted between (12) and (9) is that, with infra-marginal externalities, potential relevance need not imply Pareto relevance. The bracketed terms in (12) need not sum to zero when B is in his private utility-maximising equilibrium.

We have remained in a two-person world, with one person affected by the single activity of a second. However, the analysis can readily be modified to incorporate the effects of this activity on a multi-person group. That is to say, B's activity, Y_1, may be allowed to affect several parties simultaneously, several A's, so to speak. In each case, the activity can then be evaluated in terms of its effects on the utility of each person. Nothing in the construction need be changed. The only stage in the analysis requiring modification explicitly to take account of the possibilities of multi-person groups being externally affected is that which involves the condition for Pareto relevance and Pareto equilibrium.

For a multi-person group $(A_1, A_2, \ldots, A_n)$, any one or all of whom may be externally affected by the activity, Y_1, of the single person, B, the condition for Pareto relevance is,

$$(9A)\quad (-)\sum_{i=1}^{n} u^{A^i}_{Y_1}/u^{A^i}_{X_j} > [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]_{Y_1=\overline{Y}_1} \text{ when } u^{A^i}_{Y_1}/u^{A^i}_{X_j} < 0, \text{ and,}$$

$$\sum_{i=1}^{n} u^{A^i}_{Y_1}/u^{A^i}_{X_j} > (-)[u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j}]_{Y_1=\overline{Y}_1} \text{ when } u^{A^i}_{Y_1}/u^{A^i}_{X_j} > 0.$$

That is, the summed marginal rates of substitution over the members of the externally affected group exceed the offsetting " net " marginal evaluation of the activity by B. Again, in private equilibrium for B,

[1] This applies to the authors of this paper. For recent discussion of external effects when we have clearly intended only what we here designate as Pareto-relevant, see James M. Buchanan, " Politics, Policy, and the Pigovian Margins ", *Economica*, vol. XXVIX (1962), pp. 17–28, and, also, James M. Buchanan and Gordon Tullock, *The Calculus of Consent*, Ann Arbor, 1962.

marginal externalities are Pareto-relevant, provided that we neglect the important element involved in the costs of organising group decisions. In the real world, these costs of organising group decisions (together with uncertainty and ignorance) will prevent realisation of some " gains from trade "—just as they do in organised markets. This is as true for two-person groups as it is for larger groups. But this does not invalidate the point that potential " gains from trade " are available. The condition for Pareto equilibrium and for the infra-marginal case summarised in (11) and (12) for the two-person model can readily be modified to allow for the externally affected multi-person group.

II

The distinctions developed formally in Section I may be illustrated diagrammatically and discussed in terms of a simple descriptive example. Consider two persons, *A* and *B*, who own adjoining units of residential property. Within limits to be noted, each person values privacy, which may be measured quantitatively in terms of a single criterion, the height of a fence that can be constructed along the common boundary line. We shall assume that *B*'s desire for privacy holds over rather wide limits. His utility increases with the height of the fence up to a reasonably high level. Up to a certain minimum height, *A*'s utility also is increased as the fence is made higher. Once this minimum height is attained, however, *A*'s desire for privacy is assumed to be fully satiated. Thus, over a second range, *A*'s total utility does not change with a change in the height of the fence. However, beyond a certain limit, *A*'s view of a mountain behind *B*'s property is progressively obscured as the fence goes higher. Over this third range, therefore, *A*'s utility is reduced as the fence is constructed to higher levels. Finally, *A* will once again become wholly indifferent to marginal changes in the fence's height when his view is totally blocked out.

We specify that *B* possesses the sole authority, the only legal right, to construct the fence between the two properties.

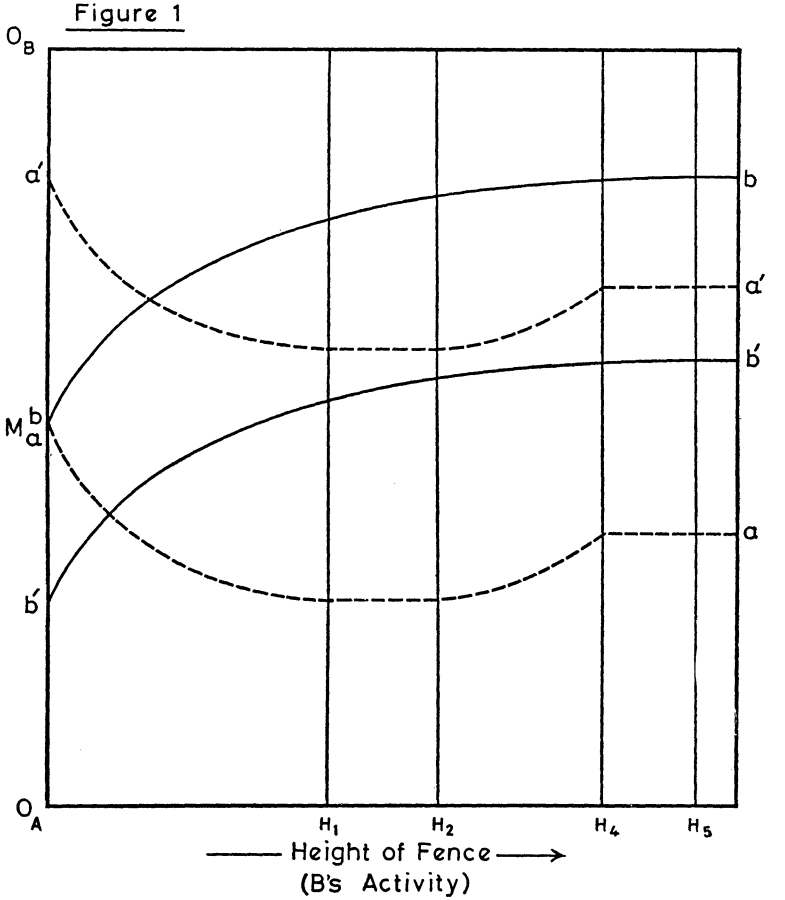
The preference patterns for *A* and for *B* are shown in Figure 1, which is drawn in the form of an Edgeworth-like box diagram. Note, however, that the origin for *B* is shown at the upper left rather than the upper right corner of the diagram as in the more normal usage. This modification is necessary here because only the numeraire good, measured along the ordinate, is strictly divisible between *A* and *B*. Both must adjust to the same height of fence, that is, to the same level of the activity creating the externality.

As described above, the indifference contours for *A* take the general shape shown by the curves *aa*, *a'a'*, while those for *B* assume the shapes, *bb*, *b'b'*. Note that these contours reflect the relative evaluations, for *A* and *B*, between money and the activity, Y_1. Since the costs of undertaking the activity, for *B*, are not incorporated in the diagram, the " contract locus " that might be derived from tangency points will

have little relevance except in the special case where the activity can be undertaken at zero costs.

Figure 2 depicts the marginal evaluation curves for *A* and *B*, as derived from the preference fields shown in Figure 1, along with some incorporation of costs. These curves are derived as follows: Assume

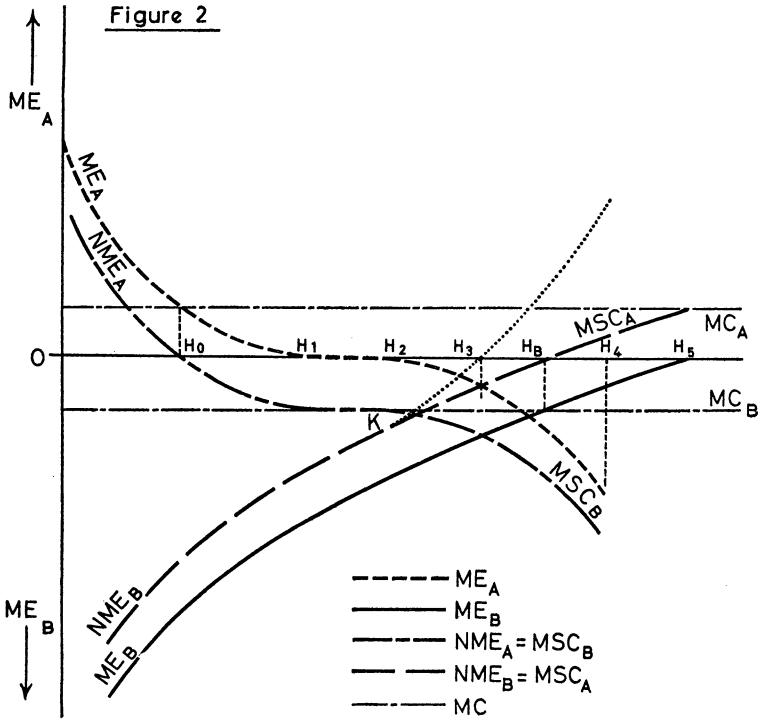
Figure 1



an initial distribution of " money " between *A* and *B*, say, that shown at *M* on Figure 1. The marginal evaluation of the activity for *A* is then derived by plotting the negatives (i.e., the mirror image) of the slopes of successive indifference curves attained by *A* as *B* is assumed to increase the height of the fence from zero. These values remain positive for a range, become zero over a second range, become negative for a third, and, finally, return to zero again.[1]

[1] For an early use of marginal evaluation curves, see J. R. Hicks, " The Four Consumer's Surpluses ", *Review of Economic Studies*, vol. XI (1943), pp. 31–41.

B's curves of marginal evaluation are measured downward from the upper horizontal axis or base line, for reasons that will become apparent. The derivation of *B*'s marginal evaluation curve is somewhat more complex than that for *A*. This is because *B*, who is the person authorised to undertake the action, in this case the building of the fence, must also bear the full costs. Thus, as *B* increases the scope of the activity, his real income, measured in terms of his remaining goods and services, is reduced. This change in the amount of remaining goods and



services will, of course, affect his marginal evaluation of the activity in question. Thus, the marginal cost of building the fence will determine, to some degree, the marginal evaluation of the fence. This necessary interdependence between marginal evaluation and marginal cost complicates the use of simple diagrammatic models in finding or locating a solution. It need not, however, deter us from presenting the solution diagrammatically, if we postulate that the marginal evaluation curve, as drawn, is based on a single presumed cost relationship. This done, we may plot *B*'s marginal evaluation of the activity from the negatives of the slopes of his indifference contours attained as he constructs the fence to higher and higher levels. *B*'s marginal evaluation, shown in Figure 2, remains positive throughout the range to the point H_5, where it becomes zero.

The distinctions noted in Section I are easily related to the construction in Figure 2. To A, the party externally affected, B's potential activity in constructing the fence can be assessed independently of any prediction of B's actual behaviour. Thus, the activity of B would,

(1) exert marginal external economies which are potentially relevant over the range OH_1;

(2) exert infra-marginal external economies over the range H_1H_2, which are clearly irrelevant since no change in B's behaviour with respect to the extent of the activity would increase A's utility;

(3) exert marginal external diseconomies over the range H_2H_4 which are potentially relevant to A; and,

(4) exert infra-marginal external economies or diseconomies beyond H_4, the direction of the effect being dependent on the ratio between the total utility derived from privacy and the total reduction in utility derived from the obstructed view. In any case, the externality is potentially relevant.

To determine Pareto relevance, the extent of B's predicted performance must be determined. The necessary condition for B's attainment of "private" utility-maximising equilibrium is that marginal costs, which he must incur, be equal to his own marginal evaluation. For simplicity in Figure 2, we assume that marginal costs are constant, as shown by the curve, MC. Thus, B's position of equilibrium is shown at H_B, within the range of marginal external diseconomies for A. Here the externality imposed by B's behaviour is clearly Pareto-relevant: A can surely work out some means of compensating B in exchange for B's agreement to reduce the scope of the activity—in this example, to reduce the height of the fence between the two properties. Diagrammatically, the position of Pareto equilibrium is shown at H_3 where the marginal evaluation of A is equal in absolute value, but negatively, to the "net" marginal evaluation of B, drawn as the curve NME_B. Only in this position are the conditions specified in (11), above, satisfied.[1]

III

Aside from the general classification of externalities that is developed, the approach here allows certain implications to be drawn, implications that have not, perhaps, been sufficiently recognised by some welfare economists.

The analysis makes it quite clear that externalities, external effects, may remain even in full Pareto equilibrium. That is to say, a position

[1] This diagrammatic analysis is necessarily oversimplified in the sense that the Pareto equilibrium position is represented as a unique point. Over the range between the "private" equilibrium for B and the point of Pareto equilibrium, the sort of bargains struck between A and B will affect the marginal evaluation curves of both individuals within this range. Thus, the more accurate analysis would suggest a "contract locus" of equilibrium points. At Pareto equilibrium, however, the condition shown in the diagrammatic presentation holds, and the demonstration of this fact rather than the location of the solution is the aim of this diagrammatics.

may be classified as Pareto-optimal or efficient despite the fact that, at the marginal, the activity of one individual externally affects the utility of another individual. Figure 2 demonstrates this point clearly. Pareto equilibrium is attained at H_3, yet B is imposing marginal external diseconomies on A.

This point has significant policy implications for it suggests that the observation of external effects, taken alone, cannot provide a basis for judgment concerning the desirability of some modification in an existing state of affairs. There is not a *prima facie* case for intervention in all cases where an externality is observed to exist.[1] The internal benefits from carrying out the activity, net of costs, may be greater than the external damage that is imposed on other parties.

In full Pareto equilibrium, of course, these internal benefits, measured in terms of some numeraire good, net of costs, must be just equal, at the margin, to the external damage that is imposed on other parties. This equilibrium will always be characterised by the strict opposition of interests of the two parties, one of which may be a multi-person group.

In the general case, we may say that, at full Pareto equilibrium, the presence of a marginal external diseconomy implies an offsetting marginal *internal* economy, whereas the presence of a marginal external economy implies an offsetting marginal *internal* diseconomy. In " private " equilibrium, as opposed to Pareto equilibrium, these net internal economies and diseconomies would, of course, be eliminated by the utility-maximising acting party. In Pareto equilibrium, these remain because the acting party is being compensated for " suffering " internal economies and diseconomies, that is, divergencies between " private " marginal costs and benefits, *measured in the absence of compensation.*

As a second point, it is useful to relate the whole analysis here to the more familiar Pigovian discussion concerning the divergence between marginal social cost (product) and marginal private cost (product). By saying that such a divergence exists, we are, in the terms of this paper, saying that a marginal externality exists. The Pigovian terminology tends to be misleading, however, in that it deals with the acting party to the exclusion of the externally affected party. It fails to take into account the fact that there are always two parties involved in a single externality relationship.[2] As we have suggested, a marginal externality is Pareto-relevant except in the position of Pareto equilibrium; gains from trade can arise. But there must be two parties to any trading arrangement. The externally affected party must compensate the acting party for modifying his behaviour. The Pigovian terminology, through its concentration on the decision-making of the acting party alone, tends to obscure the two-sidedness of the bargain that must be made.

[1] Cf. Paul A. Samuelson, *Foundations of Economic Analysis*, Cambridge, Mass., 1948, p. 208, for a discussion of the views of various writers.

[2] This criticism of the Pigovian analysis has recently been developed by R. H. Coase; see his " The Problem of Social Cost ", *Journal of Law and Economics*, vol. III (1960), pp. 1–44.

To illustrate this point, assume that A, the externally affected party in our model, successfully secures, through the auspices of the " state ", the levy of a marginal tax on B's performance of the activity, Y_1. Assume further that A is able to secure this change without cost to himself. The tax will increase the marginal cost of performing the activity for B, and, hence, will reduce the extent of the activity attained in B's " private " equilibrium. Let us now presume that this marginal tax is levied " correctly " on the basis of a Pigovian calculus; the rate of tax at the margin is made equal to the negative marginal evaluation of the activity to A. Under these modified conditions, the effective marginal cost, as confronted by B, may be shown by the curve designated as MSC_B in Figure 2. A new " private " equilibrium for B is shown at the quantity, H_3, the same level designated as Pareto equilibrium in our earlier discussion, if we neglect the disturbing interdependence between marginal evaluation and marginal costs. Attention solely to the decision calculus of B here would suggest, perhaps, that this position remains Pareto-optimal under these revised circumstances, and that it continues to qualify as a position of Pareto equilibrium. There is no divergence between marginal private cost and marginal social cost in the usual sense. However, the position, if attained in this manner, is clearly neither one of Pareto optimality, nor one that may be classified as Pareto equilibrium.

In this new " private " equilibrium for B,

(13) $u^B_{Y_1}/u^B_{Y_j} = f^B_{Y_1}/f^B_{Y_j} - u^A_{Y_1}/u^A_{X_j}$,

where $u^A_{Y_1}/u^A_{X_j}$ represents the marginal tax imposed on B as he performs the activity, Y_1. Recall the necessary condition for Pareto relevance defined in (9) above, which can now be modified to read,

(9B) $(-)\, u^A_{Y_1}/u^A_{X_j} > [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j} + u^A_{Y_1}/u^A_{X_j}]_{Y_1=\overline{\overline{Y}}_1}$, when $u^A_{Y_1}/u^A_{X_j} < 0$, and $u^A_{Y_1}/u^A_{X_j} > (-)[u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j} + u^A_{Y_1}/u^A_{X_j}]_{Y_1=\overline{\overline{Y}}_1}$, when $u^A_{Y_1}/u^A_{X_j} > 0$.

In (9B), $\overline{\overline{Y}}_1$ represents the " private " equilibrium value for Y_1, determined by B, after the ideal Pigovian tax is imposed. As before, the bracketed terms represent the " net " marginal evaluation of the activity for the acting party, B, and these sum to zero when equilibrium is reached. So long as the left-hand term in the inequality remains non-zero, a Pareto-relevant marginal externality remains, despite the fact that the full " Pigovian solution " is attained.

The apparent paradox here is not difficult to explain. Since, as postulated, A is not incurring any cost in securing the change in B's behaviour, and, since there remains, by hypothesis, a marginal diseconomy, further " trade " can be worked out between the two parties. Specifically, Pareto equilibrium is reached when,

(11A) $(-)u^A_{Y_1}/u^A_{X_j} = [u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j} + u^A_{Y_1}/u^A_{X_j}]$ when $u^A_{Y_1}/u^A_{X_j} < 0$, and $u^A_{Y_1}/u^A_{X_j} = (-)[u^B_{Y_1}/u^B_{Y_j} - f^B_{Y_1}/f^B_{Y_j} + u^A_{Y_1}/u^A_{X_j}]$ when $u^A_{Y_1}/u^A_{X_j} > 0$.

Diagrammatically, this point may be made with reference to Figure 2. If a unilaterally imposed tax, corresponding to the marginal

evaluation of A, is placed on B's performance of the activity, the new position of Pareto equilibrium may be shown by first subtracting the new marginal cost curve, drawn as MSC_B, from B's marginal evaluation curve. Where this new " net " marginal evaluation curve, shown as the dotted curve between points H_3 and K, cuts the marginal evaluation curve for A, a new position of Pareto equilibrium falling between H_2 and H_3 is located, neglecting the qualifying point discussed in Footnote 1, page 380.

The important implication to be drawn is that full Pareto equilibrium can never be attained via the imposition of unilaterally imposed taxes and subsidies until all marginal externalities are eliminated. If a tax-subsidy method, rather than " trade ", is to be introduced, it should involve bi-lateral taxes (subsidies). Not only must B's behaviour be modified so as to insure that he will take the costs externally imposed on A into account, but A's behaviour must be modified so as to insure that he will take the costs " internally " imposed on B into account. In such a double tax-subsidy scheme, the necessary Pareto conditions would be readily satisfied.[1]

In summary, Pareto equilibrium in the case of marginal externalities cannot be attained so long as marginal externalities remain, until and unless those benefiting from changes are required to pay some " price " for securing the benefits.

A third point worthy of brief note is that our analysis allows the whole treatment of externalities to encompass the consideration of purely collective goods. As students of public finance theory will have recognised, the Pareto equilibrium solution discussed in this paper is similar, indeed is identical, with that which was presented by Paul Samuelson in his theory of public expenditures.[2] The summed marginal rates of substitution (marginal evaluation) must be equal to marginal costs. Note, however, that marginal costs may include the negative marginal evaluation of other parties, if viewed in one way. Note, also, that there is nothing in the analysis which suggests its limitations to purely collective goods or even to goods that are characterised by significant externalities in their use.

Our analysis also lends itself to the more explicit point developed in Coase's recent paper.[3] He argues that the same " solution " will tend to emerge out of any externality relationship, regardless of the structure of property rights, provided only that the market process works smoothly. Strictly speaking, Coase's analysis is applicable only to inter-firm externality relationships, and the identical solution emerges only because firms adjust to prices that are competitively determined. In our terms of reference, this identity of solution cannot apply because of the incomparability of utility functions. It remains true, however,

[1] Although developed in rather different terminology, this seems to be closely in accord with Coase's analysis. Cf. R. H. Coase, *loc. cit.*

[2] Paul A. Samuelson, " The Pure Theory of Public Expenditure ", *Review of Economics and Statistics*, vol. XXXVI (1954), pp. 386–9.

[3] R. H. Coase, *loc. cit.*

that the basic characteristics of the Pareto equilibrium position remain unchanged regardless of the authority undertaking the action. This point can be readily demonstrated, again with reference to Figure 2. Let us assume that Figure 2 is now redrawn on the basis of a different legal relationship in which A now possesses full authority to construct the fence, whereas B can no longer take any action in this respect. A will, under these conditions, " privately " construct a fence only to the height H_0, where the activity clearly exerts a Pareto-relevant marginal external economy on B. Pareto equilibrium will be reached, as before, at H_3, determined, in this case, by the intersection of the " net " marginal evaluation curve for A (which is identical to the previously defined marginal social cost curve, MSC, when B is the acting party) and the marginal evaluation curve for B.[1] Note that, in this model, A will allow himself to suffer an internal marginal diseconomy, at equilibrium, provided that he is compensated by B, who continues, in Pareto equilibrium, to enjoy a marginal *external* economy.

Throughout this paper, we have deliberately chosen to introduce and to discuss only a single externality. Much of the confusion in the literature seems to have arisen because two or more externalities have been handled simultaneously. The standard example is that in which the output of one firm affects the production function of the second firm while, at the same time, the output of the second firm affects the production function of the first. Clearly, there are two externalities to be analysed in such cases. In many instances, these can be treated as separate and handled independently. In other situations, this step cannot be taken and additional tools become necessary.[2]

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[1] The H_3 position, in this presumably redrawn figure, should not be precisely compared with the same position in the other model. We are using here the same diagram for two models, and, especially over wide ranges, the dependence of the marginal evaluation curves on income effects cannot be left out of account.

[2] For a treatment of the dual externality problem that clearly shows the important difference between the separable and the non-separable cases, see Otto Davis and Andrew Whinston, " Externalities, Welfare, and the Theory of Games ", *Journal of Political Economy*, vol. LXX (1962), pp. 241–62. As the title suggests, Davis and Whinston utilise the tools of game theory for the inseparable case.