Efficiency and equalization payments in a federal system of government: a synthesis and extension of recent results

ROBIN BOADWAY and FRANK FLATTERS / Queen’s University

Abstract. This paper investigates the rationale for a system of equalization transfers in a federal system of government. The existing sources of inefficiency of resource allocation in economies with more than one level of government are synthesized in the framework of a simple decentralized multiprovince model with mobile factors of production. The sources of inefficiency arise, first because migrants respond at the margin to incorrect signals, owing to the phenomena of rent sharing and fiscal externality, and second because of global rather than local inefficiencies. The model is then extended to consider the role of the federal government in providing intergovernmental transfers in the face of such inefficiencies. In addition to synthesizing the sources of inefficiency in federal models, the equity arguments for equalization are briefly recounted.

Efficacité et paiements de péréquation dans un système fédéral de gouvernement : synthèse et extensions de certains résultats récents. Ce mémoire examine la logique qui soutend un système de paiements de péréquation dans un système fédéral de gouvernement. Les auteurs utilisent un modèle simple d’un système multi-provincial décentralisé avec des facteurs de production mobiles pour définir les sources d’inefficacité dans l’allocation des ressources dans les économies qui possèdent plus d’un niveau de gouvernement. Ces inefficacités proviennent de ce que, d’abord, ceux qui se déplacent le font en réponse, à la marge, à des signaux incorrects engendrés par le partage de la rente sur la base de la résidence et par les phénomènes d’externalités fiscales; elles sont aussi le résultat d’inefficacités globales plutôt que locales. Dans le cadre de ce modèle, les auteurs analysent le rôle du gouvernement fédéral effectuant des transferts inter-gouvernementaux pour corriger ces inefficacités. En plus de présenter une synthèse des sources d’inefficacité dans les régimes fédéraux, les auteurs font un bref rappel en passant à l’ensemble des arguments fondés sur l’équité qu’on utilise pour rationaliser les paiements de péréquation.

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Federal-provincial transfers aimed at redistributing income among provinces have been a long-standing fact of life and source of controversy in Canada. The main such program is the federal-provincial scheme of equalization payments, but there are other types of transfers between levels of government that have an implicit equalizing component (e.g., Established Programs Financing Grants).\(^1\) The question of the appropriate amount of equalization, if any, and the formula to be used have become matters of urgency for several reasons. The current equalization scheme falls due in 1982 and the renegotiation is controversial for many reasons, including the close relationship between equalization payments and the disposal of the benefits of oil and natural gas rewards.\(^2\) In addition, equalization has become a constitutional issue since the constitutional reform package embeds (albeit somewhat vaguely) the principle of equalization into the constitution.\(^3\)

There is now a sizable literature on the economics of federal states, much of it directed rather abstractly to the issue of optimal resource allocation in federal economies. We would like to extend that literature by investigating the economic rationale for intergovernmental equalizing transfers in a federal economy. More specifically, this paper attempts to do three things. First, we present a synthesis of the existing results on fiscal federalism, showing in the framework of a simple model the sources of inefficiency of resource allocation stressed in a literature. Next, using the same model we extend the analysis to consider the role for intergovernmental transfers in the face of such inefficiencies. Finally, we briefly compare these policy implications with those derived from considerations of equity. Since the equity results have been discussed at considerable length elsewhere,\(^4\) there is no need to go into their derivation in this paper.

Our analysis begins with the sorts of identical-person economies which have dominated the literature to date. Out of this will come a survey of the sorts of inefficiencies of resource allocation that can occur in economies with multiple levels of government. These models will form a basis for a discussion of the role of equalizing transfers on efficiency grounds. Subsequently, the discussion will be extended to economies of heterogeneous individuals in which equity as well as efficiency become relevant for policy issues. The analysis will be based upon simple abstract models designed to capture the basic forces at work. We believe that the general results are applicable in more complicated situations. Finally, we are

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1 The EPF grants provide equal per capita grants to the provinces financed out of federal general revenues and involve equalization to the extent that the base for federal tax collections is unevenly distributed over provinces. For a survey of existing federal-provincial transfer programs see Boadway (1980).

2 This relationship is discussed in Courchene (1980) and Helliwell (1980).

3 Also, see the Task Force on Canadian Unity (1979), the so-called Beige Paper of the Quebec Liberal Party (1980) and the Trudeau government’s Charter of Rights (1981). There is a question as to whether, even if equalization turns out to be justified on economic grounds, it is an appropriate obligation to include the constitution (contrary views appear in Boadway and Norrie, 1980 and Usher, 1981). We shall ignore that question.

4 Boadway and Flatters (1982).
ignoring the sort of inefficiency in federal economies arising out of interjurisdictional spillovers of benefits and costs. The literature on this topic is well established, non-controversial and of no essential relevance for the issue of equalization.\(^5\)

**MODELS OF FEDERAL ECONOMIES WITH IDENTICAL INDIVIDUALS**

Though often formulated in simplistic terms, models of federal economies or systems of local government have proven to be powerful tools for elucidating some of the inherent inefficiencies induced by interjurisdictional factor mobility on the one hand, and decentralized public sector decision-making on the other. We shall begin with the simplest of models in order to concentrate on the first of these issues, especially the inefficiency of individual labour migration decisions in models of local government. Local governments (hereafter referred to as provinces) will be assumed to behave myopically with respect to the size of their populations. This turns out to be an innocuous assumption, since, as shown in Boadway (1982) and Lange (1982), local governments cannot improve themselves by behaving otherwise in these models. That is, provinces are assumed to behave as if their populations were given, despite the fact that they are not.

Ideally, we would like our model to incorporate several features of the real world, including interregional trade and factor mobility, differences in local resource endowments, differences in industrial structure, heterogeneity of tastes, income and labour force quality, complicated federal and provincial tax/tariff/subsidy structures, unemployment, and the provision of public services at several levels with possibly some spillovers of benefits. However, to begin with it is useful to abstract from many of these features of regional economies in order to concentrate upon the interaction between labour mobility, decentralized public sector decision-making and differences in regional productivity or resource endowments. The tendencies to inefficiency in these simple models will continue to hold in more complex economies. Subsequent analysis will involve a gradual relaxation of some of these assumptions.

**Federalism models with myopic provincial governments**

Let us begin with a simple federation consisting of two provinces. The federation has a given aggregate population \(N\), which also represents the labour supply. It is assumed to be costlessly mobile between provinces. In equilibrium \(L_1\) will locate in province 1 while \(L_2\) will locate in 2. Each province has a Ricardian-type aggregate production function of the form \(f^p(L_i)\), where \(f''(L_i) > 0\) and \(f''(L_i) < 0\) (at least beyond some level of \(L_i\)). The output thus produced can be allocated to use in the private sector or in the public sector within the province. Thus, the marginal rate of transformation (MRT) between private goods and public sector goods is unity. This assumption is not essential. It is easy to generalize the model to allow MRT to vary.

All individuals are endowed with one unit of labour and have identical preferences.

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\(^5\) Surveys of this topic may be found in Oates (1972) and Boadway (1979).
represented by the utility function \( u(X_i, G_i/L_i^\alpha) \) where \( X_i \) is the per capita consumption of private goods, \( G_i \) is the quantity of public sector goods provided in province \( i \), and \( G_i/L_i^\alpha \) is the services of the latter. The parameter \( \alpha \) is an index of the 'publicness' of provincial public services and can take values \( 0 \leq \alpha \leq 1 \). This formulation allows us to span the spectrum from pure public goods to pure private goods since these correspond to \( \alpha = 0 \) and \( \alpha = 1 \) respectively. The public service would be 'impure' or 'partially rivalrous' to use Musgrave's (1969) term if \( \alpha \) is between zero and unity. Assuming initially that the value of output of a province accrues entirely to its own residents, per capita private goods consumption \( X_i \) is given by \( f_i(L_i) - G_i/L_i \). Notice that we are ignoring any activity by the central or federal government.

Provincial government behaviour is easy to characterize in this simple model. Each one is assumed to behave myopically in the sense that it ignores the influence of its actions on migration. This assumption has been used almost universally in the literature on local public goods, and as we have mentioned, it does not affect the results of the model. Under this assumption, a provincial government that aims at maximizing the welfare of its citizens will solve the following problem:

\[
\text{Max } u[f(L) - G/L, G/L^\alpha],
\]

where we shall follow the convention of dropping the provincial subscripts when analysing the behaviour of a representative province.

The first-order condition for this unconstrained maximization problem yields:

\[
L^{(1-\alpha)} u_G/u_X = 1,
\]

where \( u_G \) and \( u_X \) are the marginal utilities of \( G/L^\alpha \) and \( X \) respectively. If \( \alpha = 0 \), this corresponds to the familiar Samuelson condition that the sum of the marginal rates of substitution (MRS) equals the MRT (here unity). Similarly, if \( \alpha = 1 \) it reduces to the optimality condition for pure private goods. The solution to (2) for \( G \) determines how provincial output is divided between private and public sectors. To characterize a migration equilibrium, we must examine how per capita utility in a province varies with population size.

6 This is similar to the technique used by Borcherding and Deacon (1972) and Bergstrom and Goodman (1973) in their empirical studies of local public expenditures in the U.S.A. Both papers found \( \alpha \) close to unity. An alternative way to introduce impurity into public goods is to explicitly include congestion costs. See Buchanan and Goetz (1972), Flatters, Henderson, and Mieszkowski (1974), and Oakland (1972) for examples of this approach. The former method is more convenient for our purposes but both yield qualitatively similar results.

7 The structure of this model is similar to that found in much of the theoretical literature on the subject, except for our generalization to allow for impure public goods. See, for example, Flatters, Henderson, and Mieszkowski (1974), Stiglitz (1977), and the survey in Atkinson and Stiglitz (1980). Other models have used slightly different devices from the diminishing marginal product of labour when applied to fixed land to generate diseconomies of population increase. Buchanan and Goetz (1972) use club goods (see also Henderson, 1974), Starrett (1980) used spatial considerations. In these alternative models the qualitative results derived in this section continue to hold.

8 Exceptions to this are Boskin (1973), Pauly (1973), and the recent paper by Starrett (1980).
Migration equilibrium

The maximum $u$ attained in problem (1) is contingent upon $L$, which is exogenously determined for each province. The relationship between per capita utility and provincial population size is defined by the maximum value function $V(L)$ associated with problem (1):

$$V(L) = \max_G u[L - G/L, G/L^\alpha].$$

This gives the per capita utility attained under various values of $L$ when the province is maximizing myopically. From the envelope theorem\(^9\) we know that $dV/dL = \partial u/\partial L$ (evaluated at the optimum); therefore, using (2)

$$V'(L) = u_X(f'(L) - X - \alpha G/L)/L$$

$$V''(L) = [u_{XX}(f'(L) - X)/L - u_{GX}\alpha G/L^\alpha]V'(L)/u_X$$

$$+ u_X(f''(L) + \alpha G/L^2)/L - V'(L)/L.$$  \(5\)

From (4) we observe that the optimal population is that at which the marginal product of labour equals per capita consumption of the private good plus $\alpha G/L$\(^10\). This is intuitively plausible, since the contribution of a marginal person to production is $f'(L)$, while his additional claim on resources is per capita private consumption $X$ and his congestion cost imposed on others is $\alpha G/L$\(^11\). From (5), when $G$ is a pure public good ($\alpha = 0$) and provided $V'(L) > 0$ at $L = 0$, we can deduce that since the first two terms on the right-hand side are negative the graph of $V(L)$ is single-peaked with $V''(L)$ eventually becoming positive as in figure 1\(^12\). This diagram reflects the coexisting effects of diminishing returns to labour and economies of scale in the consumption of the public good. Inspection of (5) indicates that this single-peakedness of $V(L)$ is also plausible when $\alpha < 1$, although one could contrive cases in which there are more than one peak. For simplicity we shall assume single-peakedness, although our analysis remains intact under more complicated shapes of $V(L)$.

Free migration ensures that labour allocates itself among provinces until $V(L)$ is equalized. This is shown in figures 1 and 2, where subscripts 1 and 2 refer to provinces 1 and 2. Figure 1 shows the case in which there is a unique stable equilibrium. On the other hand, in figure 2 there are three possible equilibria with

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\(^9\) A convenient summary of the use of maximum value functions and the envelope theorem may be found in Dixit (1976).

\(^10\) This characterization assumes that the optimal population is positive and finite. In addition one requires that $V(L)$ be strictly concave as discussed below. This condition on optimal population is similar to that obtained by Flatters, Henderson, and Mieszkowski (1974), generalized here to include congestion costs.

\(^11\) The interpretation of $\alpha G/L$ as the marginal congestion costs comes from noting that the change in the utility benefit from public services as $L$ increases is $-\alpha u_G G/L^\alpha + 1$. Converting this to private goods units using (2) and multiplying by $L$ to aggregate over the entire population yields $-\alpha G/L$.

\(^12\) $V''(L)$ may be positive initially if $f''(L) > 0$ at low levels of labour supply. Eventually, however, diminishing marginal product of labour will occur and $V''(L) < 0$. 

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FIGURE 1  A unique equilibrium

FIGURE 2  Multiple equilibria
only the corner ones being stable. The internal equilibrium \( L' \) is unstable since any departures from it would result in the complete depopulation of one or other of the provinces. The ultimate consequence of this could be a level of \( V \) which is lower than that attained at \( L' \). This result is reminiscent of Myrdal’s (1957) cumulative causation model of regional disparities, but here it is generated from neo-classical postulates alone.\(^\text{13}\)

One way of distinguishing case 1 from case 2 is the size of the federation’s total population relative to the optimal amount. If each region had its optimal population (i.e., that which maximized its \( V \)), the aggregate \( N \) would be less than that which actually exists in 1 and more than that which exists in 2.\(^\text{14}\) In that sense it is overpopulated federations which would tend to have a unique stable equilibrium (although even it could be unstable). In an underpopulated federation an internal equilibrium, if it existed, would tend to be unstable.\(^\text{15}\) Since it is hard to imagine Canada as being anything but underpopulated, this seems to bode ill for the stability of the migration process.

We do not observe the complete depopulation of provinces or regions, however, and that is presumably because there is not free migration as we have so far postulated. It is instructive to illustrate the role of migration costs in lending stability to the model. In the free migration model the condition for stability of the internal equilibrium can be written:

\[
\frac{dV_1(L_1)}{dL_1} - \frac{dV_2(N - L_1)}{dL_1} < 0
\]

or

\[(V'_1 + V'_2) < 0.\] (6)

Let us now introduce migration costs. If the initial distribution of population is such that migration will go from 2 to 1, the migration equilibrium condition can be written:

\[V_1(L_1) = V_2(L_2) + m(L_2),\] (7)

where \( m(\cdot) \) is the migration cost function and is assumed to depend upon \( L_2 \) (and hence upon the number of migrants). A stable internal equilibrium now requires that:

\[(V'_1 + V'_2 + m') < 0.\] (8)

\(^{13}\) See Kaldor (1970) for another view of regional disparities stressing cumulative causation.

\(^{14}\) Actually, the definition of optimal population for a federation is an ambiguous concept. An equilibrium in which each region had its optimal population would generally be unsustainable since the maximum \( V(L) \) would be higher in one region than another. This is incompatible with free migration. The optimal population under free migration (with \( V \)s equated) might make more sense, and the text could be interpreted in that way. Even this is not totally satisfactory, since, as we shall see, migration is generally not efficient.

\(^{15}\) There are other patterns than those we have drawn. For example, cases of multiple internal equilibria are depicted in Atkinson and Stiglitz (1980). Such multiple equilibria can occur in both under- and overpopulated cases. In addition, an underpopulated federation can have a stable free migration equilibrium that is unique if the optimal \( V \)s are different. We have benefitted from discussions with Sam Wilson and some of his unpublished work on the stability of free migration equilibria.
Even if \((V_1' + V_2') > 0\) at the equilibrium, this condition might still hold if \(m'(L_2) < 0\); that is, if per capita migration costs rise with the number of migrants (sufficiently rapidly). This is a reasonable assumption, since it is consistent with the notion that migrants have differing degrees of attachment to their home province. Figure 2 illustrates this case. The allocation \(L^0\) is the historically-given initial allocation of labour over provinces while \(L^m\) is the after-migration equilibrium (which is stable). The introduction of migration costs has precluded complete depopulation. However, it is also important to observe that \(L^m\) is not in any sense an efficient allocation of labour. On the contrary, as the diagram indicates, the residents of both regions may well be ultimately worse off as a result of migration.

More complicated analyses would look at the development over time of the federation as the total population grows. We shall not pursue that topic here. Rather, we shall indicate another source of inefficiency that has been prominent in the literature. Let us revert to the stable internal solution case with free migration. In this case an equilibrium such as \(L^e\) in figure 1 is achieved where equal per capita utilities are obtained in the two provinces. Even in this stable case it is unlikely that the distribution \(L^e\) is optimal. The most important reason for this non-optimality has to do with the inefficiency of the migration process itself. Owing to a type of migration externality, the private signals that potential migrants are getting differ from the social signals they should be receiving.16

**Inefficiency of migration**

Consider the migration equilibrium \(L^e\) in figure 1. We can obtain an expression for the net benefit to the existing residents of a province from having one more resident. For example, in province 1 this would be (using equation (4) and the definition of \(X\)):

\[
L_1V_1'(L_1) = u_X(f_1'(L_1) - f_1(L_1)/L_1 + (1 - \alpha)G_1/L_1)
\]

\[
= u_X((1 - \alpha)G_1/L_1 - R_1/L_1),
\]

where \(R_1 = f_1(L) - L_1f_1'(L_1)\) is the total rent in province 1, assuming wage rates to be equal to marginal products. Dividing through by \(u_X\) we obtain the benefit to existing residents of an additional migrant in terms of consumption of \(X\), denoted \(MB_{L_1}\),

\[
MB_{L_1} = (1 - \alpha)G_1/L_1 - R_1/L_1.
\]  

(9)

A similar expression holds for province 2.

This may be interpreted as the total amount of \(X\) residents of 1 would be willing to give up to have an additional migrant, given that the migrant were to receive his per capita share of rents. \(MB_{L_1}\) consists of two parts. The first, \((1 - \alpha)G_1/L_1\), is the per capita tax payment less congestion cost for each resident in province 1 and will be called the *fiscal externality*. An additional resident consumes the public good \(G\) at an

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16 Another reason why free migration is non-optimal is that a social welfare optimum in a federal model, even with identical individuals, typically requires that utilities be unequal for otherwise identical individuals. This is clearly inconsistent with free migration models. The phenomenon was first pointed out by Mirrlees (1972), and is discussed in the context of local public goods by Hartwick (1980).
opportunity cost equal to the congestion he imposes on others, but at the same time contributes his share to the financing of \( G_1 \) thus reducing the tax bill of existing residents. The newcomer’s tax contribution \( T_1 \) is just his per capita tax payment, \( G_1/L_1 \). The second term is the per capita share of the rent generated in province 1 assumed, at this stage, to be equally distributed among residents. The more residents there are, the smaller is the share of the rent going to each person. A marginal resident reduces the rents of existing residents by \( R_1/L_1 \).

Assuming, as would be true at the free migration equilibrium, that the marginal migrant is indifferent between residing in 1 and 2, the net benefit to the nation from his moving from 2 to 1 is given by:

\[
NB = MB_{L_1} - MB_{L_2} = \left( \frac{(1 - \alpha)G_1}{L_1} - \frac{(1 - \alpha)G_2}{L_2} \right) - \left( \frac{R_1}{L_1} - \frac{R_2}{L_2} \right). \tag{10}
\]

There is no reason that these should sum to zero in the free migration equilibrium \( L^e \), except fortuitously. It is worth considering the two components of the right hand side of (10) in turn.

*Fiscal externality* / This model has typically been formulated in the literature for pure public goods (\( \alpha = 0 \)). In this case the fiscal externality term will vanish only if per capita tax payments are identical in both provinces. As Flatters, Henderson, and Mieszkowski (1974) show, this requires that the compensated elasticity of demand for public goods be exactly unity, an unlikely eventuality. As (10) indicates, however, the force of the fiscal externality can be diluted if, instead of being pure public goods, provincial public services are impure or congested. The closer \( G \) is to being private, the nearer \( \alpha \) is to unity, and the smaller the fiscal externality is. It disappears altogether if \( G \) is purely private (\( \alpha = 1 \)). We shall follow the convention of calling \( G \) a *quasi-private good* in this case.

*Rent sharing* / Provincial per capita rent sharing as a source of inefficiency has played a prominent role in the recent Canadian literature on regional policy.\(^{17}\) It is also analytically similar to the phenomenon of sharing the returns to public capital discussed by Usher (1977), although sharable rents can be captured on private capital as well as on public capital via provincial taxation policies. The point is simply that if workers get a share of provincial rents solely on the basis of residency, they respond to their average product rather than to their marginal product (wage rate) in migration decisions. The force of this argument depends upon the rents accruing publicly and being disbursed to individuals on the basis of residency alone. If all rents went to persons solely on the basis of ownership of the rent-generating factor the inefficiency would disappear, since residency and ownership of property would be independently determined. In principle, even with the collectivization of rents and free migration, this source of inefficiency in the allocation of labour (as well as the fiscal externality) can be considerably lessened if there exists another fixed factor required for residency into which rents can be capitalized (e.g., residential land). It is hard to

\(^{17}\) See, for example, Courchene and Melvin (1980), Flatters and Purvis (1980), Helliwell (1980), and Eden (1981).
believe that this is a significant deterrent to long-term migration in Canada where residential land is relatively abundant in the rent-rich provinces.

The upshot of this discussion of migration in a very simple federal economy is that free migration will generally lead to an inefficient allocation of labour over the federation. This inefficiency can be eliminated by a particular system of interregional transfers of private goods either voluntarily arranged by the provinces or imposed by the central government. Inappropriate grants would lead to inefficiency. The appropriate grants would ensure that \( NB = 0 \) at the optimum. In the simple model we have been analysing so far it can be shown that the size of the total transfer from 1 to 2, denoted \( S \), would satisfy the following equation:\(^{18}\)

\[
\frac{G_1(1 - \alpha) - R_1 + S}{L_1} = \frac{G_2(1 - \alpha) - R_2 - S}{L_2}.
\]

or, solving for \( S \),

\[
S = \frac{L_1L_2}{L_1 + L_2} \left[ \frac{G_2(1 - \alpha)}{L_2} - \frac{G_1(1 - \alpha)}{L_1} \right] + \left( \frac{R_1}{L_1} - \frac{R_2}{L_2} \right).
\]

That is, the size of the transfer would be an average of the provinces’ MBSs each weighted by the other provinces’ population; or, proportional to the differences in fiscal externality and per capita rents. This is the equalization formula called for on efficiency grounds in this model. Equalization formulas in more general models will be introduced as our analysis proceeds.

Even in this simple model it is worth being cautious about these policy results. The formula for \( S \) in (12) is based upon marginal optimality conditions which may not correspond to a global maximum. We saw earlier that, especially if the federation is underpopulated, the migration equilibrium may not be near the globally efficient point, owing to inherent instabilities. In this case much more informationally demanding total analyses are required.

We chose our model to be as simple as possible in order to illustrate the nature of migration inefficiency. However, the basic point continues to be valid in more complicated settings. Adding migration costs to the model does not affect the net benefit expression (10). Because of migration costs, however, the utility levels will tend to differ from one region to another, being lower in the province experiencing out-migration. These differences in utility can be used to motivate interprovincial transfers on equity grounds.

The introduction of heterogeneity of individuals also complicates matters, and analyses in this area are by no means complete. The fiscal externality and rent sharing problems remain, but the characterization of the free market equilibrium depends critically upon the technology and upon the manner in which local governments

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\(^{18}\) These formulas are derived from the following central government problem (see Hartwick (1980)):

\[
\max_{L_1, S} u^1[(f^1(L_1) - G_1 - S)/L_1; - G_1/L_1^a]
\]

subject to \( u^1 = u^2 \).
finance public expenditures. There is a natural tendency for ‘likes’ to wish to congregate together, as in the literature on club goods. In the latter, if the production side is abstracted from (by giving everyone exogenous incomes) and if financing is by benefit taxation, one expects to see ‘likes’ congregate together.19 This tendency is easily countered, however, by complementarities among the sorts of labour supplied by various persons, or by general forms of taxation. For example, Wheaton (1975) has argued that only under poll taxation will homogeneous communities tend to be stable. A survey of these results takes us too far afield.

It is reasonably easy, however, to extend our model to include other factors of production, such as capital, and heterogeneity of individuals, so long as tastes are similar. We shall do that below. Also, we shall allow the provincial governments some leeway in the choice of their tax instruments for financing public services. These modifications will have important consequences for the appropriate form of interregional transfers.

Introduction of capital and heterogeneous individuals

Capital / We begin by allowing for capital as well as for labour and resources in the production technology, by treating the economy as being a decentralized market economy with private factor ownership, and by allowing the provinces considerably more scope in choosing their tax mix. Consequently, we assume aggregate production functions for each province which take the form \( f(L, K) \) where \( K \) is capital and both \( K \) and \( L \) are perfectly mobile over provinces.20 There is, in addition, a fixed factor (e.g., land, resources, etc.) which we do not include explicitly, since it is in fixed supply to each province and immobile, but which receives the residual income after labour and capital have been paid. Labour and capital are assumed to obtain their marginal products, and the residual going to the fixed factor will be termed the rent.21 The rent could differ from one province to the next because of either differing factor supplies or differing technologies or productivities. We need not be too explicit in explaining the source of regional disparities. Provinces are taken to be price-takers and outputs are normalized to have unit prices as before.

We consider the special case in which all persons are identical and perfectly mobile between provinces. In addition, each person is assumed to own the same proportion of the nation’s resources. In particular, each person owns one unit of labour and a portion \( \beta = 1/N \) of the nation’s capital and fixed factors, regardless of where he resides. We are thus assuming away foreign ownership. Different assumptions could be made, of course, but at the expense of some simplicity. Provincial government \( i \) is assumed to provide a pure public good \( G_i \) and to finance it by proportional taxes on

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19 See the discussion in McGuire (1974) and Berglas (1976).
20 Instead of using an aggregate production function we could have followed the trade literature and adopted, say, a two-sector Heckscher-Ohlin-Samuelson model, possibly amended to include resources as in Copithorne (1979). This elaboration of the production side is not undertaken, since it does not affect the qualitative results derived here.
21 It is natural to think of this rent as accruing to natural resources, but the term is much more general. It applies to any factor fixed in supply to a province. It could also include social capital as analysed by Usher (1977).
labour income at the rate \( t_i \), on property income at the rate \( m_i \), on locally generated rents at the rate \( x_i \), and on local capital returns at the rate \( z_i \). Thus, \( t_i \) and \( m_i \) may be thought of as residence-based taxes while \( x_i \) and \( z_i \) are source-based taxes.

Individuals obtain utility from the private good and the provincial public good. The per capita consumption of the private good in province \( i \) is determined by after-tax income:

\[
X_i = f_{L}^{i}(1 - t_i) + \beta(1 - m_i)[R_i(1 - x_i) + R_2(1 - x_2) + f_{K}^{i}(1 - z_1)K_1 + f_{K}^{2}(1 - z_2)(K - K_1)],
\]

where \( R_i \) is the rent generated in province \( i \) and, by assumption, is given by:

\[
R_i(L_i, K_i) = f_{L}^{i}(L_i, K_i) - L_i f_{LL}^{i}(L_i, K_i) - K_i f_{KL}^{i}(L_i, K_i).
\]

Note that \( \partial R_i/\partial L_i = -L_i f_{LL}^{i} - K_i f_{KL}^{i} \) and \( \partial R_i/\partial K_i = -K_i f_{KK}^{i} - L_i f_{KL}^{i} \). These are of ambiguous sign. We have also assumed that the total stock of capital in the nation is fixed at \( K \) and the return to capital is determined endogenously. An alternative approach would have been to allow foreign capital to come in at a given world rate of return. The analysis is similar in this case.

Provincial government expenditures are financed entirely by taxes in this ‘real’ economy. The government budget constraint is given by:

\[
G_i = t_i f_{L}^{i} L_i + m_i [P] L_i \beta + x_i R_i + z_i f_{K}^{i} K_i,
\]

where the \( P \) denotes total property income across provinces and is given by the term in square brackets in (13). Substituting (15) into (13) we obtain the expression for private goods expenditures with the government budget implicitly accounted for:

\[
X_i = f_{L}^{i} + \beta P + x_i R_i / L_i + z_i f_{K}^{i} K_i / L_i - G_i / L_i.
\]

Notice that \( t_i \) and \( m_i \) have disappeared from this formulation. The reason is as follows. First, \( t_i \) and \( m_i \) are essentially perfect substitutes in this model. Both are taxes levied on the basis of residency and, other than influences on the migration decision, there are no disincentive effects associated with them (e.g., labour supply is fixed). Next, from the point of view of (15) and (16), these residency-based taxes are essentially budget-balancing items that disappeared when (15) was substituted into (13). In the following analysis, the amount of residency-based tax collected is that residually required to balance the budget. Whether \( t_i, m_i \) or some combination is used is irrelevant. One could also interpret the general retail sales tax as equivalent to a uniform income tax in this model.

Equilibrium in a two-province economy will be characterized by the simultaneous satisfaction of a migration equilibrium condition and a capital market equilibrium condition. Under free migration the former is given by:

\[
u_1[f_{L}^{1} + \beta P + x_1 R_1 / L_1 + z_1 f_{K}^{1} K_1 / L_1 - G_1 / L_1, G_1] = u_2[f_{L}^{2} + \beta P + x_2 R_2 / (N - L_1) + z_2 f_{K}^{2} (K - K_1) / (N - L_1) - G_2 / (N - L_1), G_2].
\]

where \( L_1 \) and \( K_1 \) are the labour and capital supplies in province 1, and the residuals
$N - L_1$ and $K - K_1$ are factor supplies in province 2. The capital market equilibrium requires that the net-of-capital-tax return to capital be the same in both provinces, or:

$$f_K/(1 - z_1) = f_K/(1 - z_2).$$

These two equations include two unknowns, $L_1$ and $K_1$. If there is a unique stable equilibrium, and we have already seen how unlikely this is in underpopulated regions in the absence of migration costs, then we could solve (17) and (18) for $L_1$ and $K_1$ as functions of the tax and expenditure variables.

A complete characterization of equilibrium also requires a description of provincial tax and expenditure policies. At the theoretical level there appear to be conflicting influences at work. On the one hand, the literature on tax competition among local governments suggests that provinces have an incentive to undertake inefficient tax cutting or the provision of subsidies in order to attract factors of production. On the other hand, the theory of tax incidence suggests that governments have an incentive to overexpand since part of the burden of marginal tax collections will be shifted to non-residents. This is also at the basis of a recent argument by Starrett (1980) that local governments may undertake greater than optimal public expenditures in order to attract labour inefficiently. A complete investigation of this subject would lead us somewhat astray from the main point of our analysis, so we shall simply point to some general conclusions that have emerged from the analysis of the recent literature.

The first point is that there seems to be no general presumption that strategic behaviour of provincial governments will tend to cause them to overspend or underspend on public goods to attract labour or capital. In a quite general model of this sort Boadway (1982) has shown that it is optimal for provincial governments to follow the Samuelson rule for the provision of public goods – a province can do no better by behaving strategically than by behaving myopically in this regard. The effects of strategic behaviour on the choice of tax policies are more ambiguous. The choice of taxes on rents and capital income (both source-based taxes) will depend on the balance of two conflicting forces. First, the fact that they are partially incident on non-residents; second, working in the opposite direction, is the fact that they will tend to induce immigration from other regions, thus diluting the existing residents’ share of publicly collected rents and income from taxes on capital. In the case of capital, there is the additional consideration that increases in capital taxes will cause the province to lose capital.

From a social point of view, we are interested in comparing the equilibrium

22 See Boskin (1973) and Pauly (1973). See also the discussion of source-based tax competition in Helliwell (1977).

23 This would be an implication of the theory of interjurisdictional tax shifting as discussed, say, in McLure (1970).

24 He argues that this will be the case if local governments finance expenditures by direct taxation on residents. The incentive is alleged to be the other way, however, if property taxation is used. The analysis of Boadway (1982) takes issue with those results.

25 See Boadway (1982) and Lange (1982) for more complete analyses. A referee has pointed out to us that the discussion of taxation and public goods is paralleled by the analysis of education and taxation in the brain drain literature. For a treatment of this topic see, for example, Bhagwati (1976).
interprovincial allocation of resources with the efficient allocation of resources. One can show that the allocation of resources that maximizes per capita utility, given that free migration exists, will be characterized by the following:

\[ L_i u_{G_i} - u^i = 0 \quad i = 1, 2, \]  
\[ f_K^1 = f_K^2, \]  
\[ f_L^1 = f_L^2 - X_2. \]  

The first of these is just the Samuelson condition for public goods. We have already indicated that both provinces will satisfy this if behaving optimally. Furthermore, they will continue to behave optimally if the public good is impure in the sense discussed earlier.

The second condition, equality of the marginal products of capital, is required for production efficiency. From (18) we see it will be satisfied only if \( z_1 = z_2 \). However, \( z_1 \) and \( z_2 \) are determined independently by each province, and it would only be by chance that provinces choose \( z_1 = z_2 \). Therefore, the efficient capital allocation condition is unlikely to be satisfied.

Finally, condition (21) is similar to that obtained in our simple model for optimal labour allocation. It states essentially that labour ought to be allocated over provinces in such a way that the social benefit of having an additional worker in each region is equalized. An alternative form of expression is to substitute for \( X_1 \) and \( X_2 \) from (16) into (21) to yield:

\[ \frac{G_1}{L_1} - \frac{x_1 R_1}{L_1} - \frac{z_1 f_K^1 K_1}{L_1} = \frac{G_2}{L_2} - \frac{x_2 R_2}{L_2} - \frac{z_2 f_K^2 K_2}{L_2}. \]  

This optimality expression has a similar interpretation to that of our earlier model (see the discussion following equation (10), above). The first term \( G_i/L_i \) represents the fiscal externality. In the more general case of impure public goods it becomes \( G_i(1 - \alpha)/L_i \) so that if \( \alpha = 1 \), the fiscal externality disappears. The second term represents the rent-sharing term as before, while the last term represents per capita capital tax collections. Even if \( x_i \) and \( z_i \) are identical in the two provinces, these terms will not disappear (unless they equal zero). In general, there is no reason to believe that equation (22) will be satisfied by provinces’ independently choosing \( x_1 \) and \( z_1 \). It might be noted as well that the residence-based tax rates \( t_i \) and \( m_i \) do not appear in this efficiency condition. That will turn out to be of some significance in discussing appropriate federal government policy.

In order to achieve national efficiency in interprovincial resource allocation, the federal government would need two policy instruments to correct for the two distortions arising from decentralized public sector decision-making. The first

26 The problem is to maximize \( u(X_1, G_1) \) subject to \( u(X_1, G_1) = u(X_2, G_2) \) and \( L_i X_1 + (N - L_1) X_2 + G_1 + G_2 = f^i(L_1, K_1) + f^i(N - L_1, K - K_1) \).

27 Furthermore, if there were perfect mobility of capital internationally, \( z_1 \) and \( z_2 \) would have to be zero for efficiency.
distortion is that arising from interprovincial differences in taxes on capital (lack of fulfillment of equation (20)). This could be difficult to eliminate, except by imposing a discriminatory tax on capital income in the province with the lower capital tax rate, \( z_i \) so as to equalize the marginal products of capital in the two provinces. An alternative would be to impose or come to some agreement on a scheme of capital tax harmonization.

The second policy instrument could be a system of transfers from the residents of one province to those of the other, so that equality (21) would be satisfied. In the general case of impure public goods the aggregate size of the transfer from 1 to 2 would be such as to satisfy:

\[
\frac{G_1(1 - \alpha) - x_1R_1 - z_1f_K^1K_1 + S}{L_1} = \frac{G_2(1 - \alpha) - x_2R_2 - z_2f_K^2K_2 - S}{L_2}.
\]

Or, equivalently,

\[
S = \frac{L_1L_2}{L_1 + L_2} \left\{ \left( \frac{G_2(1 - \alpha)}{L_2} - \frac{G_1(1 - \alpha)}{L_1} \right) + \left( \frac{x_1R_1}{L_1} - \frac{x_2R_2}{L_2} \right) + \left( \frac{z_1f_K^1K_1}{L_1} - \frac{z_2f_K^2K_2}{L_2} \right) \right\}.
\]

The latter equation is similar to (12) derived in our simpler model. The only differences are: (1) the addition of a new term representing the differences in per capita collections of capital taxes, and (2) the modification of the rent differential term to indicate that it is only publicly collected rents (in this case through rent taxes) that create distortion in interprovincial migration.

The ‘equalization’ transfer could be thought of as consisting of three parts. The first is the difference in the fiscal externality between the two provinces. This is not readily measurable since \( \alpha \) is not observable. Such evidence as we have would suggest that \( \alpha \) is close to unity, in which case this term would disappear. That leaves the latter two terms, the difference between provincial per capita tax collections on rents and on capital. We shall refer to these terms together as the per capita source-based tax collections. National efficiency considerations would dictate that these actual tax collections be fully equalized. Note that it is not the tax capacities that should be equalized but actual taxes collected.

There are several difficulties associated with pursuing full equalization according to (24), even ignoring the fiscal externality term. Most of them revolve around the constitutional issue of how much independent tax raising power the provinces ought to have. The greater is the power of the provinces to choose their preferred tax mix, the greater will be the efficiency cost of a federal system of government as opposed to a unitary one. Under full equalization, the provinces would have little incentive to levy taxes on capital or rents at all, since these revenues would be equalized over provinces anyway. This would indeed be an efficient solution, especially with respect to capital. If our model had internationally mobile capital at given interest rates, efficiency would dictate having \( z_i = 0 \). The case with rents is more suspect. For one
thing, aside from the interprovincial migration effects, rent taxes are efficient taxes and we would not wish to remove all incentive to use them by the province. Also, a constitution that in fact gives full ownership of resources to the provinces would inevitably be in conflict with a scheme of equalizing those rents despite the efficiency costs of not equalizing. At some point the conflict between decentralization and efficiency must be resolved. As we have indicated elsewhere (Boadway and Flatters, 1982), considerations of equity help us to find a resolution of this issue.

So far our analysis concludes that there is no need to equalize residence-based taxes; aside from the fiscal externality, it is only interprovincial differences in per capita source-based tax collections that need be equalized to achieve efficiency. The reason that residence-based tax collections are not included in our equalization scheme is simple to explain. Residence-based taxes are simply payments from provincial residents to themselves. Since all persons in a province are assumed to be identical, all pay the same residence-based taxes and all receive an equivalent amount back in the form of provincial government services. Therefore, residence-based taxes do not affect interprovincial migration under our assumptions. As soon as we allow for a heterogeneous labour force and some redistributive behaviour of provincial governments in their tax-expenditure decisions, however, residence-based taxes will affect interprovincial migration and will enter into the equalization formula. We turn to this analysis in the next section.

**Heterogeneous labour** / We now assume that workers differ in their income-earning abilities and that provincial governments pursue redistributive budgetary activities; it is no longer the case that (aside from source-based taxes which we assumed to finance equal per capita services) taxes paid by each resident of a province are used to finance public services of equal value. As we shall see, the exact nature of these redistributive activities will determine the ideal equalization scheme. Therefore, it is important to start with some specific assumptions.

We begin by assuming that residence-based taxes in each province are proportional to each individual’s income, while public services, assumed to be of a quasi-private nature, are distributed on an equal per capita basis. The net effect of the provincial fiscal structures is therefore progressive.

The real income for a citizen includes both the real value of private goods consumed and of publicly provided goods and services. Retaining our fixed price assumption, the real income for citizen \( j \) in one of the provinces may be written:

\[
Y_j = (1 - r)p_{Ij} + g_j,
\]  

(25)

where \( p_{Ij} \) is his personal income from wages and property \( (f_{Ij} + p_j) \), \( g_j \) is his public sector benefits, and \( t \) is the proportional income tax rate. Under our assumptions about the provincial fiscal structure, per capita public services, \( g \), may be written

\[
g = r\bar{p} + xR/L + zf_kK/L,
\]  

(26)

where \( \bar{p} \) is per capita personal income (from wages and property) in the province.
Substituting this into the expression for individual real income, real income may be written:

\[ Y_j = \pi_j + t(\bar{p}_1 - \pi_j) + xR/L + zf_K K/L. \]  

(27)

The last three terms represent the person’s net fiscal benefits (NFBS) from provincial fiscal activities in his province of residence. This NFBS from provincial activity now comprises two parts: per capita source-based tax collections and the net gain through redistributive income-tax-financed services (the term in \( t \)). For a person of below average \( \pi_j \), this latter term is positive, and vice versa.

Now compare residents of provinces 1 and 2 who have identical personal incomes (\( \pi_j^1 = \pi_j^2 \)). Their real incomes will differ owing to differences in the NFBS from provincial activity. Assuming both provinces have the type of fiscal structure just described,

\[ Y_j^1 - Y_j^2 = (x_1 R_1 + z_1 f_K^1 K_1)/L_1 - (x_2 R_2 + z_2 f_K^2 K_2)/L_2 
\]  

\[ + (t_1 \bar{p}_1 - t_2 \bar{p}_2) + \pi_j(t_2 - t_1). \]  

(28)

The first two terms represent the difference in per capita source-based tax revenues. We have already dealt with the appropriate (efficient) equalization scheme to deal with them. The next term reflects the fact that a higher per capita level of provincial services can be financed at a given tax rate in a high income province. To the extent that the high income province chooses to lower its tax rate this advantage will be diminished. On the other hand, the latter term shows that an individual at a given income level will benefit through lower tax payments, if the fiscal advantage is exploited through lower taxes.

To consider the efficiency case for equalization in this model, suppose now that labour is costlessly mobile among provinces. The real income of person \( j \) is given by (27) and this will be equalized over provinces by migration. Since gross property income of a migrant is independent of province of residence, migration equilibrium will be given by:

\[ f_L^1 + t_1(\bar{p}_1 - \pi_j^1) + (x_1 R_1 + z_1 f_K^1 K_1)/L_1 = f_L^2 + t_2(\bar{p}_2 - \pi_j^2) 
\]  

\[ + (x_2 R_2 + z_2 f_K^2 K_2)/L_2. \]  

(29)

Efficiency in the allocation of labour will be given by \( f_L^1 = f_L^2 \). Equation (29) is not compatible with efficiency since the NFBS are typically not equal; efficiency can be assured only if differences in NFBS over provinces are eliminated. Even if these interprovincial differences in per capita source-based tax collections were eliminated, there still would be distortionary differences in NFBS, owing to residence-based taxes (the last two terms in (28) or the second term on each side of (29)). Suppose, for instance, that both provinces levied the same personal tax rates (\( t_1 = t_2 \)). The NFBS difference due to residence-based taxes would be simply \( t(\bar{p}_1 - \bar{p}_2) \) and would represent the difference in per capita public sector benefits arising solely from differences in residence-based tax bases. Notice that the NFBS difference is identical.
over all income groups. Therefore the equalization program that is called for on efficiency grounds is one that fully equalizes per capita revenues from both source-based taxes and residence-based taxes.

This system of equalization has the following characterization. If provinces behave identically ex post, they will all provide the same level of public services at the same tax rates. Any differences in the ability to provide public services at given tax rates will be eliminated by the equalization system. Our initial assumption that all provinces levy the same rate of tax in the absence of equalization is, in fact, unnecessary. The formula used in actual equalization schemes uses ex post tax rates and so provincial behaviour in the absence of equalization is irrelevant. Of course, if provinces behave differently ex post, no system of equalization can achieve perfect efficiency. The most that can be achieved is that equalizing both source and residence-based tax revenues will give provinces the potential to provide the same level of services at the same tax rates, though not all will conform. Full equalization in this context would presumably use national average tax rates (like the present system).

It should be noted, of course, that our results on equalization of residence-based taxes are sensitive to the way in which provincial budgets are formulated. We have used a special case in which provincial redistribution comes about through proportional residence-based taxation and equal per capita benefits, and this may not be far from the truth. It is reasonably straightforward to deduce the sorts of equalization schemes that would be called for under different patterns of provincial redistribution, and we shall not do so here. Suffice it to say that if provincial governments are more (less) redistributive than we have assumed, a greater (less) degree of equalization is called for.

CONCLUDING REMARKS

In this paper we have investigated some of the efficiency implications of models of fiscal federalism for the design of interprovincial equalization schemes. Several key points have emerged from the analysis. First, one cannot expect in general that migration decisions taken by individuals in a decentralized federal economy will lead to an efficient allocation of labour over provinces. Not only can the migration process be locally inefficient (in the sense of not satisfying the first-order social efficiency conditions), it may well also be globally inefficient. Second, self-interested provincial governments acting on behalf of their residents have an incentive to take budgetary actions that, from a national point of view, lead to inefficiencies and inequities. Finally, the federal government faced with the inefficiencies and inequities arising out of individual and provincial government behaviour will be justified in using a system of equalization payments as a policy instrument in the pursuit of nationwide equity and efficiency.

The specific form the equalization formula should take will depend upon the manner in which the economy is presumed to operate. Our analysis was carried out for a particular set of assumptions. Prices were assumed fixed, provinces were
assumed to provide equal per capita benefits of a quasi-private sort to residents, and residence-based taxes were assumed to be incident on residents in proportion to income. Furthermore, provinces were assumed to behave in similar ways. Under these assumptions, the ideal equalization scheme from an efficiency point of view is one that fully equalizes all provincial tax revenues per capita.

As we have shown elsewhere (Boadway and Flatters, 1982), on equity grounds, full equalization would also be desirable if the federal government took the view that horizontal equity demanded that persons in identical positions before provincial and federal government budgets should be so afterwards. Full equalization eliminates differences in NFBS over provinces and allows vertical equity to be achieved via the progressive income tax. The other view of horizontal equity is that the federal government should take as its starting point the post-provincial government allocation of real incomes. If so, equalization need be applied only to a proportion \( \tau \) of source-based tax revenues, where \( \tau \) is the overall average federal marginal tax rate. No equalization of income tax revenues would be called for. Notice that this scheme is neither efficient nor horizontally equitable in the above broader sense.

The implications of other assumptions about the economy for equalization could easily be worked out using the arguments of this paper. The variability of output and input prices could be allowed. This would require a careful consideration of incidence effects of various taxes. Other methods of getting at the rents of provincial resources could (and should) be included in the equalization formula. These include the passing on of ‘rents’ to provincial residents by lower prices (as hydro utilities may do), and the collection of rents as profits of Crown corporations. As already mentioned, other provincial government redistributive assumptions could be made. Other provincial taxes could be explored more explicitly, such as indirect taxes and property taxes.

It ought to be remarked that our analysis is of a long-run nature. That is to say, we are comparing allocations of resources in situations in which the labour market is in equilibrium. In practice, there may be two difficulties with this approach. First, the allocation of labour observed in the absence of equalization may be out of equilibrium in the sense that, say, labour in a ‘have-not’ region may not have had time to adjust to the opportunities in the ‘have’ region. The process of migration takes time and our analysis only partly captures that element in our cost of migration term \( m \). If this is the case, a scheme of equalization will, on the one hand, slow down the path of migration from the have not region, but, on the other, will ensure that the path will take the economy to the correct long-run allocation. Second, if the labour market is in equilibrium to begin with, albeit an inefficient one, the introduction of equalization will cause a move to a new equilibrium only after a period of adjustment. Our analysis does not incorporate the dynamics of adjustment of the labour market from one equilibrium to another. We would conjecture, however, based on existing analyses of adjustment costs (e.g., Mussa, 1978), that the introduction of dynamic considerations would not affect the policy prescription for equalization based on long-run analysis.

Finally, virtually all our discussion of equalization concerns the correction of marginal inefficiencies of migration. As discussed in the second section, the
migration process is liable to be globally inefficient as well. Future research could fruitfully consider the implications for intergovernmental transfers of global inefficiency in underpopulated economies.

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