

Incentive-Compatible Debt Contracts: The One-Period Problem

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In a simple model of borrowing and lending with asymmetric information we show that the optimal, incentive-compatible debt contract is the standard debt contract. The second-best level of investment never exceeds the first-best and is strictly less when there is a positive probability of costly bankruptcy. We also compare the second-best with the results of interest-rate-taking behaviour and consider the effects of risk aversion. Finally we provide conditions under which increasing the borrower's initial net wealth must reduce total investment in the venture.

1. INTRODUCTION

A great deal of effort has recently gone into the study of *implicit contract models* (ICM) in an attempt to explain the failure of competitive labour markets to generate efficient levels of employment. (See Hart, 1983, for a survey of this work and further references). Relatively little attention has been given to the theoretical analysis of capital market imperfections, a phenomenon we believe to be at least as important as labour market imperfections when it comes to explaining the inefficient use of resources, including "unemployment". In this paper we seek to fill this gap by analysing a model of credit contracts between firms that need outside finance in order to operate efficiently on the one hand and the institutions that provide the money on the other.

We were led to this problem by our current interest in the question of *what is the firm's budget constraint?* In economies with complete markets, of course, this question does not arise in a meaningful form. But when markets are incomplete the firm does face a budget constraint of some sort at each date and the form of this constraint may crucially affect its behaviour. There is no intrinsic virtue in a budget constraint which requires the firm to repay all debts with probability one and, in fact, most legal systems allow for some insurance, in the form of bankruptcy, against low income states. Perhaps the central message of our work is simply that the question of the firm's budget constraint should be formulated as a contract problem. And under certain conditions, the form of the optimal budget constraint is the same as the standard debt contract with bankruptcy. But there is more to the analysis than that. We are naturally interested not only in the *shape* of the firm's budget constraint but also in its *position*. In other words, is there credit-rationing?

Asymmetric information plays a crucial role in our model as it does in the ICM. The revenue of a firm depends both on investment in inputs and on the state of nature. The firm can observe the state of nature directly at no cost; other agents cannot. However,

unlike the ICM, agents who are initially uninformed may become informed by paying a positive fee which may depend on investment or the state. The combination of asymmetric information and the costs of observing the state result in inefficiency even though agents are risk neutral. In the ICM risk-aversion is needed to generate inefficiency. (But note that we rule out the possibility of "negative consumption" for firms; this restriction is quite natural and without it the analysis would be both trivial and uninteresting.) The final major difference between the ICM and our model is the ordering of decisions and information. In the ICM the state is observed before labour is employed. In our model investment takes place before the state is observed. The level of investment is necessarily state-independent whereas in the ICM employment is typically state-contingent. The impact of asymmetric information in our model falls directly on the distribution of revenue between borrower and lender and only indirectly on the level of investment.

The model can be formally interpreted simply as a description of risk-sharing with asymmetric information and positive observation costs. But a more interesting interpretation is to treat the act of observation as "bankruptcy". This interpretation is interesting for two reasons. The first is that the optimal contract turns out to be the standard debt contract under which the state is observed if and only if the firm cannot repay the loan in full. That is, the state is observed only if the firm is insolvent. The second is that bankruptcy does involve a transfer of information. Before the receiver can pay the creditors of a bankrupt firm he must ascertain exactly what the firm is worth. This corresponds, in our model, to discovering the true value of the state s .

If the act of observing s is interpreted as forcing the firm into bankruptcy the costs of observation are going to be substantial. The scrap value of the firm may be much less than its value as a going concern. In order to maintain the firm as a going concern a new owner with appropriate managerial skills must be found. This will take time and during that time mistakes will be made, valuable employees lost, goodwill and reputation damaged. For all these reasons, bankruptcy is a costly business. But bankruptcy has a beneficial side: without it contracts would be even more constrained and welfare would be even lower.

We obtain four main results in this paper. *First*, as mentioned above, we can show that under certain conditions the optimal credit contract takes the form of a standard debt contract with bankruptcy, where "bankruptcy" is identified with the act of observing the state. By a standard debt contract we mean a contract which requires a fixed repayment when the firm is solvent, requires the firm to be declared bankrupt if this fixed payment cannot be met and allows the creditor to recoup as much of the debt as possible from the firm's assets. The first property follows directly from an incentive-compatibility argument and the fact that the creditor cannot observe the state when the firm is not bankrupt. The third property holds because repaying as much as possible in bankruptcy states allows the fixed repayment in non-bankruptcy states to be minimized, thus minimizing the probability of bankruptcy and hence the costs. The second property is a bit more subtle than the other two. Roughly, it depends on showing that switching from bankruptcy to non-bankruptcy in a solvent state requires no costly changes in the rest of the contract. It is worth noting that the proof of the optimality of the standard debt contract follows from the definition of the problem. To prove the converse requires a number of non-trivial assumptions.

Second, we examine conditions under which credit-rationing occurs. The optimal (second-best) investment level never exceeds and typically falls short of the first best. This is the basic underinvestment result. Other types of credit-rationing are found in the literature. Credit-rationing can take the form of denying credit to some firms entirely (cf.

Stiglitz and Weiss, 1980). In the capital markets we consider this sort of rationing cannot occur. A firm denied credit by one institution will simply go to another and eventually it will obtain a loan. Rationing the size of the loan is, in any case, what we are after. It generalizes the analysis in Gale (1983*a*, 1983*b*) which uses a no-bankruptcy constraint to generate Keynesian effective demand failures. The results obtained here show that this kind of effective demand failure can occur even if the firm's budget constraint is derived from an optimal contract which admits bankruptcy.

Third, we compare optimal contracts with the result of interest-rate-taking behaviour. We confirm the well known proposition that equilibrium debt contracts will usually involve credit-rationing in the sense that the optimal loan and interest rate are both smaller than they would have been as the result of interest-rate-taking behaviour of the debtor. However, this result has nothing to do with the discrepancy between the first-best level of investment and the optimal contractual level. Even the first-best investment level may fall short of the investment level under interest-rate-taking behaviour.

Our *fourth* result concerns the effect of changes in liquidity, as measured by the firm's initial net wealth, on the optimal level of investment. Lack of liquidity in this sense lies at the root of the credit-rationing problem because, if the firm's net wealth were large enough to finance the first-best investment, the firm would obviously choose that level of investment. Intuition therefore suggests that as the firm's liquidity declines so does the optimal investment, i.e. the credit-rationing problem gets worse. But this is not necessarily the case. In fact, we present conditions under which the relationship between liquidity and investment *cannot* be monotonic. The case we look at is special, being intended only as an illustration, but the much fuller analysis of this question in Gale and Hellwig (1984*a*) indicates that it is by no means untypical.

In contrast to our analysis, most previous work on credit and bankruptcy has taken the form of the credit contract as given. An exception is Diamond (1984) who derives the optimal contract under similar informational assumptions to ours. But since he makes the costs of bankruptcy *endogenous*, the optimal contract leads to the first-best investment level. This is because the penalties on the bankrupt firm are equivalent to perfect bond-posting and the problem of bankruptcy becomes innocuous.

Townsend (1978) considers the effect of costly observation of the state on optimal implicit contracts but not in the context of credit markets.

The recent work by Stiglitz and Weiss (1980, 1981) adopts an approach which is superficially similar to ours. It is worthwhile considering the differences in their assumptions and results. We assume that asymmetric information concerns the observation of revenue; Stiglitz and Weiss, on the other hand, assume that asymmetric information concerns the choice of project (safe or risky). In our model, the riskiness of the venture is determined by the level of investment. We have chosen our informational assumptions because they generate the standard debt contract. Asymmetric information about the level of investment would be hard to analyse and would not produce a recognizable debt contract. Stiglitz and Weiss do not actually derive the form of the optimal contract in their model; more precisely, they consider an example which is so simple that the form of the contract is virtually dictated by the structure. The projects are of fixed size so there is no investment decision. The revenue function assumes only two values, one positive, the other zero, so the repayment schedule is also determined. Thus, most of the questions we analyse cannot be raised in the Stiglitz-Weiss framework. On the other hand, they study (in Stiglitz and Weiss, 1980) the role of credit-rationing as an incentive mechanism in long term contracts. Roughly, there is shown to be a role for refusing credit completely (conditional on poor performance) to enforce the choice of the correct

project. Again, it is not clear how robust these results are to changes in the very simple structure of their model, in particular the all-or-nothing nature of the investment decision. In a later paper (Gale and Hellwig, 1984b) we study a two-period contract problem and find that with divisible investments it is generally not optimal to have complete exclusion.

Our credit-rationing result depends on a number of assumptions but, if there is a central assumption, it appears to be the assumption of diminishing returns to investment.² Given the fixed opportunity cost of investment, diminishing returns to investment ensure that as the level of investment increases beyond some point the distribution of profits shifts to the left. The point at which this shift starts to occur is less than the first-best level of investment. (This is partly a result of diminishing returns and partly a result of the structure of bankruptcy costs). Thus reducing investment some way below the first-best level reduces the probability of bankruptcy and hence the costs of bankruptcy. In a competitive market, diminishing returns are necessary in the neighbourhood of the first best so one might conjecture that our result holds under some more general specification of costs and revenues. But we have not investigated this.

The model we have studied in this paper is, by the standards of contract theory, a fairly general one. Although the type of asymmetric information we choose to study leads to a simple form of the optimal contract, the analysis, especially of comparative static properties, quickly becomes quite complicated. The harder part of the analysis concerning the structure of the efficient set of contracts and the effect of longer term relationships between borrower and lender is left for future papers. What we present here is intended as a basis for that later work. The rest of the paper is organized as follows. Section 2 contains a description of the model, a formal statement of the problem and a characterization of the *form* of the optimal contract. In Section 3 we study the optimal (second-best) level of investment, comparing it with the first best and with the result of interest-rate-taking behaviour. In Section 4 we indicate how the analysis changes if entrepreneurs are assumed to be risk-averse. In Section 5 we discuss the relationship between the entrepreneur's net wealth and the level of investment.

2. STATEMENT OF THE CONTRACT PROBLEM

The contracts we study are written in a competitive capital market. The market comprises two types of economic agents, *investors* and *entrepreneurs*. Entrepreneurs wish to undertake risky ventures but lack the necessary resources so they turn to the investors for external finance. Investors are meant to be banks or other deposit-taking, financial institutions. They are assumed to hold sufficiently large and diversified portfolios of investments to achieve *perfect risk-pooling*. Then investors will behave as if they are *risk-neutral*.³ Furthermore, they can obtain deposits by paying the rate of interest on riskless securities. In a competitive capital market each investor treats the rate of interest on riskless securities as a parameter and assumes he can obtain whatever funds he needs at that rate. This rate of interest, henceforth denoted by $i > 0$, is therefore the (fixed) opportunity cost of funds for the investor.

Because investors are effectively risk-neutral and have unlimited access to funds at the riskless rate of interest, we can assume without loss of generality that each entrepreneur obtains funds from at most one investor. By the same token, each investor can determine an optimal contract with one entrepreneur independently of his dealings with other entrepreneurs. In what follows, therefore, we consider only a single, representative, investor-entrepreneur pairing. The capital market is competitive because it contains a large number of individually insignificant investors and entrepreneurs. Competitive

pressure will lead each investor-entrepreneur pair to write a contract which maximizes the expected utility of the entrepreneur subject to the constraint that the expected return to the investor covers the opportunity cost of funds. It is clear this constraint must be satisfied; otherwise the investor is worse off than if he had not entered into the contract. On the other hand, if the expected utility of the entrepreneur is not maximized subject to this constraint, some other investor can offer a contract which is more attractive to the entrepreneur and still make a profit at the going rate of interest.

Each entrepreneur is the owner-manager of a single firm. We can think of investment as an input to a production process which yields a random revenue, either because of technological uncertainty or because of uncertainty about the price at which the output will be sold. The production process involves two dates, indexed 0 and 1. At the first date, the level of investment is chosen; at the second, the revenue is observed. At date 0 the entrepreneur's *net wealth* (ignoring the value of the firm as a going concern) is $W_0 = A_0 - R_0$, where $A_0 \geq 0$ is the entrepreneur's liquid assets and $R_0 \geq 0$ the firm's initial indebtedness. The entrepreneur is assumed to be *risk-neutral*; he maximizes the expected value of his "wealth" at date 1. The returns to the risky venture are described by a revenue function f . An investment of l units at date 0 produces a revenue of $f(s, l)$ units in state s at date 1.

Assumption 1. $f: \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable⁴ on the interior of $\mathbb{R}_+ \times \mathbb{R}_+$ and continuous at the boundary. For any $(s, l) \gg 0$:

- (i) $f(0, l) = f(s, 0) = 0$;
- (ii) $D_2 f(s, l) > 0$, $D_{22} f(s, l) < 0$ and $D_{12} f(s, l) > 0$.

The probability distribution of states⁵ is represented by a cumulative distribution function H with support in \mathbb{R}_+ .

A crucial assumption is that agents have asymmetric information. At date 0, neither the investor nor the entrepreneur can observe the state. At date 1, the entrepreneur observes the state free of charge. The investor can observe the state only at some cost. In fact, costs may be imposed on the entrepreneur as well as the investor. This makes sense in terms of our interpretation of the event of observing the state as "bankruptcy". In that case there may be some penalty imposed on the debtor for going bankrupt. Let $c_0 \geq 0$ be the fixed cost imposed on the entrepreneur when the investor observes the state. The cost born by the investor is assumed to be a function of both the state and the level of investment. In the bankruptcy interpretation, the major cost born by the creditor is the failure to recover the full revenue $f(s, l)$, and this will generally be a function of s and l . Let $c_1(s, l)$ denote the observation cost for the investor in state s with investment l .

Assumption 2. $c_1: \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable on the interior of $\mathbb{R}_+ \times \mathbb{R}_+$ and continuous at the boundary. For any $(s, l) \gg 0$, $D_2 c_1 \geq 0$, $D_{22} c_1 \leq 0$ and $D_1 c_1 \geq 0$.

It is important to understand the distinction between the costs imposed on the respective agents. To the extent that the observation of the state by the investor involves purely pecuniary costs, their distribution between the two agents is immaterial. This is because the investor only receives a return sufficient to cover the opportunity cost of his investment, so all other costs are effectively born by the entrepreneur. Non-pecuniary costs (penalties) are different, however, because unlike pecuniary costs they are not limited

by the entrepreneur's ability to pay. In what follows c_0 will always be treated as (the pecuniary equivalent of) a non-pecuniary cost. All other costs of observing the state are lumped in with $c_1(s, l)$.

When an investor and an entrepreneur get together to write a contract they have four decisions to make. First, they must choose $l \geq 0$, the investment in the risky venture. Second, they must decide how the total financing requirement $l + R_0$ is to be divided between them. If the entrepreneur contributes $C_0 \geq 0$ the investor will be required to contribute $L = l + R_0 - C_0$. We can regard C_0 as equity and L as the new loan. The total financing requirement is $l + R_0$ since the existing liability must either be repaid or rolled over. Both of these decisions are made at date 0 and so are independent of the state. The third decision is how the revenue is to be divided in each state of nature. Let $W_1(s)$ denote the entrepreneur's final wealth in state s and $C_1(s)$ the investor's return from the venture in state s . Both are net of the costs of observing the state of nature. The fourth and final decision is which states are to be observed by the investor. This decision is represented by a random variable B which takes the value 1 if a state is to be observed and 0 otherwise. A *contract* is defined to be an array (l, C_0, C_1, W_1, B) .

In the absence of asymmetric information or, what amounts to the same thing, costs of observing the state, there would be no obstacle to achieving the first-best level of investment. This is the investment, denoted by l^* , which maximizes the expected profit from the venture:

$$l^* \in \arg \max_{l \geq 0} E[f(s, l) - (1+i)l].$$

From Assumption 1 it is clear that l^* , if it exists, is unique. Even in the presence of asymmetric information it may be possible to achieve the first-best if, for example, the entrepreneur's assets are sufficient to finance the first-best investment without the assistance of the investor. This case is not interesting, so we rule it out.

Assumption 3. $0 < l^* < \infty$ and $W_0 = A_0 - R_0 < l^*$.

Because observation of the state is normally costly, there is no reason to expect the investor to observe the state at all times. For that reason the implementation of the contract requires the entrepreneur to reveal the state to the investor. But the contract will only be carried out as intended if the entrepreneur reveals this information truthfully and he will only do so if he has no incentive to do otherwise. In other words, the contract must be incentive-compatible; it must give the entrepreneur no incentive to lie. The potential for deception is limited by two factors. First, if the entrepreneur falsely announces a state s in which $B(s) = 1$ he is sure to be found out because the contract requires the investor to observe the state. Thus we need only worry about announcements of states for which $B(s) = 0$. Second, in order for the investor to be deceived he must receive the same income as if the announced state had actually occurred. It must be feasible for the entrepreneur to pay this income from actual revenues and that places another constraint on the entrepreneur's ability to lie.

Let \hat{s} be the true state and let s be some other state such that $B(s) = 0$. The entrepreneur's wealth, if he announces s , is denoted by $W(s, \hat{s})$ and defined by

$$W(s, \hat{s}) = f(\hat{s}, l) + (1+i)(A_0 - C_0) - C_1(s).$$

The announcement of s is *feasible* if and only if $W(s, \hat{s}) \geq 0$. The contract specifies that the entrepreneur's "wealth" (including the cost of observation) should be $W_1(\hat{s}) - B(\hat{s})c_0$.

Therefore the contract is *incentive-compatible* if and only if

$$\begin{aligned} &\text{for any states } s \text{ and } \hat{s} \text{ such that } B(s) = 0 \text{ either} \\ &W(s, \hat{s}) < 0 \text{ or } W(s, \hat{s}) \leq W_1(\hat{s}) - c_0 B(\hat{s}). \end{aligned} \tag{1}$$

Proposition 1. *A contract (l, C_0, C_1, W_1, B) is incentive-compatible if and only if*

- (a) *there exists a constant R_1 such that $C_1(s) = R_1$ whenever $B(s) = 0$ and*
- (b) *for any states s and \hat{s} such that $B(s) = 0, B(\hat{s}) = 1$ and $C_1(s) \leq f(\hat{s}, l) + (1+i)(A_0 - C_0), R_1 - C_1(s) \geq c_0 + c_1(s, l)$.*

The proof is immediate from Definition (1). Condition (a) is fairly obvious: if C_1 is not constant across the non-observed states the entrepreneur has an incentive, whenever the value of C_1 is high in a non-observed state, to announce a non-observed state with a lower value. This must be feasible. If the true state is an observed state and it is feasible for the entrepreneur to lie, condition (b) says that the return to the investor must at least absorb the cost of observation. Otherwise the entrepreneur would rather pay a higher value of R_1 to avoid the costs of observation. Thus, although on average observation costs are born by the entrepreneur, at the margin they appear to be born by the investor. In the sequel, we deal only with incentive-compatible contracts.

The *optimal contract problem* is to choose an incentive-compatible contract to maximize the entrepreneur’s expected utility subject to the investor’s zero-profit condition. Formally, the problem is to choose a contract (l, C_0, C_1, W_1, B) to solve:⁶

$$\max E[W_1 - c_0 B] \tag{2i}$$

$$\text{s.t. } EC_1 \geq (1+i)(l + R_0 - C_0) \tag{2ii}$$

$$C_1 + W_1 \leq f - c_1 B + (1+i)(A_0 - C_0) \tag{2iii}$$

$$0 \leq l, \quad 0 \leq C_0 \leq A_0, \quad 0 \leq W_1 \tag{2iv}$$

$$(l, C_0, C_1, W_1, B) \text{ is incentive-compatible.} \tag{2v}$$

(2ii) is the zero-profit condition in inequality form; (2iii) is a feasibility condition; (2iv) is self-explanatory but note that $0 \leq W_1$ is crucial if the problem is to be non-trivial; and (2v) is the incentive-compatibility condition.

The rest of this section is devoted to characterizing the *form* of the *optimal contracts*, that is, solutions to (2). The first step is to simplify the objective function and some of the constraints. Although (2ii) and (2iii) have been written as inequalities they are in fact equations at an optimum. (This is not entirely obvious but we leave the proof to the reader.) Use equation (2iii) to obtain an expression for W_1 which can be substituted in (2i). Then substitute the expression for EC_1 obtained from equation (2ii) in (2i). The objective function is now

$$E[f - (1+i)l - (c_0 + c_1)B] + (1+i)W_0.$$

Since the last term is a constant it can be dropped without affecting the solution to the problem. Finally, using equation (2iii) we can express the inequality $W_1 \geq 0$ in terms of other variables. Then W_1 has been eliminated from the problem and (2iii) can be dropped as well. The optimal contract problem can be rewritten as

$$\max E[f - (1+i)l - (c_0 + c_1)B] \tag{3i}$$

$$\text{s.t. } EC_1 = (1+i)(l + R_0 - C_0) \tag{3ii}$$

$$C_1 \leq f + (1+i)(A_0 - C_0) - c_1 B \quad (3iii)$$

$$0 \leq l, \quad 0 \leq C_0 \leq A_0 \quad (3iv)$$

$$(l, C_0, C_1, B) \text{ is incentive-compatible.} \quad (3v)$$

Condition (3ii) corresponds to the old condition $W_1 \geq 0$. Note that we are entitled to drop the reference to W_1 in (3v) since it does not appear in the conclusion of Proposition 1.

In order to characterize the form of the optimal contract we need two important concepts. The first is *maximum equity participation* (MEP). A contract (l, C_0, C_1, B) exhibits MEP if and only if $C_0 = A_0$. The second is the *standard debt contract* (SDC). A debt contract in the usual sense has three essential features. When the debtor is solvent a debt contract involves a fixed payment to the creditor. The debtor is declared bankrupt if and only if he cannot make this fixed payment and, in the event of bankruptcy, the creditor recovers as much as he can. A contract (l, C_0, C_1, B) is said to be a SDC if and only if

$$(\text{fixed repayment}) \text{ for some } R_1, (1-B)(C_1 - R_1) = 0 \quad (4i)$$

$$(\text{bankruptcy decision}) B = 1 \Leftrightarrow f < R_1 \quad (4ii)$$

$$(\text{maximum recovery}) BC_1 = B(f - c_1). \quad (4iii)$$

Here we have identified the event $B = 1$ with bankruptcy and interpret $f - c_1$ as the recoverable part of the firm's revenue after bankruptcy. These two concepts can be shown to characterize the form of the optimal contract though MEP is not strictly necessary. In fact the degree of equity participation turns out to be immaterial. To see this let (l, C_0, C_1, B) be an optimal contract and define a new contract (l, C'_0, C'_1, B) by choosing $0 \leq C'_0 \leq A_0$, $C'_0 \neq C_0$ and putting $C'_1 = C_1 - (1+i)(C'_0 - C_0)$. It is clear that the new contract satisfies the feasibility conditions (3iii) and (3iv) and the zero-profit condition (3ii). From inspection of Proposition 1 it can be seen that it is incentive-compatible as well. Since C'_0 and C'_1 do not enter directly in the objective function (3i) the new contract must be optimal. This argument shows that any level of equity can be optimal so that MEP can never be a *necessary* condition for optimality. We only have the weaker result that:

Proposition 2. *Any optimal contract is weakly dominated by a contract with MEP.*

The indeterminacy of C_0 is a consequence of the fact that, in order to motivate the SDC, we have allowed agents complete freedom in writing the terms of their contract, including the possibility of using liquid assets $(1+i)(A_0 - C_0)$ to repay the "debt" at date 1. In terms of the contracting problem set out above this makes perfect sense. But if we wished to take the bankruptcy interpretation more seriously we might insist that, in the event of bankruptcy, the liquid assets $(1+i)(A_0 - C_0)$ would be protected from seizure by limited liability (A_0 is the *entrepreneur's* asset position; R_0 the *firm's* indebtedness). The incentive-compatibility constraints would be more complicated but other things being equal, one would expect R_1 to be higher because repayment in bankruptcy states would be lower. This would provide a positive incentive to put $C_0 = A_0$ in order to reduce R_1 and the probability of bankruptcy.

In what follows only contracts with MEP are considered. Let (l, C_0, C_1, B) be an optimal contract and suppose it has MEP. For some fixed but arbitrary value of R'_1 define

a new contract (l, C_0, C'_1, B') by putting

$$B' = \begin{cases} 0 & \text{if } f \geq R'_1 \\ 1 & \text{if } f < R'_1 \end{cases}$$

and

$$C'_1 = \begin{cases} R'_1 & \text{if } B' = 0 \\ f - c_1 & \text{if } B' = 1. \end{cases}$$

Suppose that $R'_1 = R_1$ where R_1 is the constant value of C_1 when $B = 0$. Whenever $B = B'$, feasibility (3iii) and the construction of C'_1 imply that $C'_1 \geq C_1$. If $B' < B$ incentive-compatibility ensures that $C_1 \leq R_1 \leq C'_1$. Since feasibility implies $B' \leq B$ this proves $C'_1 \geq C_1$ when $R'_1 = R_1$. So we can choose R'_1 to satisfy the zero-profit constraint (3ii). (Choose the smallest value if there is more than one). By construction the new contract is feasible ((3iii) and (3iv)). Incentive-compatibility is satisfied by any SDC with MEP. Then (l, C_0, C'_1, B') is an admissible contract and since, by an earlier argument, $B' \leq B$ it must be optimal.

Proposition 3. *Any optimal contract is weakly dominated by a SDC with MEP.*

To obtain the converse we need to assume that observation costs are positive and responsive to small changes in the contract.

Proposition 4. *Let (l, C_0, C_1, B) be an optimal contract with MEP and let R_1 be the value of C_1 when $B = 0$. For any $\epsilon > 0$ suppose that*

$$\int_{R_1 - \epsilon \leq f < R_1} (c_0 + c_1) dH > 0.$$

Then (l, C_0, C_1, B) is a SDC.

Proof. Let (l, C_0, C'_1, B') be the SDC derived from (l, C_0, C_1, B) as described earlier. Since both contracts are optimal

$$E(B - B')(c_0 + c_1) = 0.$$

Since $B' \leq B$ the hypothesis in the proposition implies that $R'_1 = R_1$, otherwise the costs associated with (l, C_0, C'_1, B') would be strictly less. Then if $B \neq B'$ they must differ on a non-null set of states on which $f \geq R_1$. But Assumption 2 and our hypothesis imply that $E(B - B')(c_0 + c_1) > 0$, a contradiction. Therefore $B = B'$. Feasibility (3iii) and the zero-profit constraint imply that $C'_1 = C_1$. \parallel

In the rest of the paper we deal only with SDCs with MEP. These contracts can be characterized by the two parameters l and R_1 . More precisely, given an ordered pair (l, R_1) one can derive a unique contract (l, C_0, C_1, B) by putting $C_0 = A_0$ and using (4) to define (C_1, B) . A slightly different representation proves more convenient. Note first of all that under our assumptions

$$R_1 \geq 0 \quad \text{for any optimal contract.} \tag{5}$$

If $R_1 < 0$ then incentive-compatibility implies that $C_1 < 0$ and the zero-profit condition then implies $l - W_0 < 0$. Then it is easy to see that $l = l^*$, contradicting Assumption 3. We can also assume without essential loss of generality that

$$R_1 \leq \sup \{f(s, l) \mid H(s) < 1\} \quad \text{for any optimal contract.} \tag{6}$$

To see this, note that if $R_1 > f(s, l)$ with probability 1 then $C_0 = A_0$ and the feasibility constraint (3iii) imply that $B \equiv 1$. Then the value of R_1 is immaterial.

In view of (5) and (6) and Assumption 2 there exists a state γ such that $f(\gamma, l) = R_1$. If there is more than one such then let γ be the smallest. Then the contract (l, R_1) can equally well be represented by the ordered pair (l, γ) . Conversely any ordered pair (l, γ) defines a contract if we put $R_1 = f(\gamma, l)$. We call γ the *bankruptcy point* for obvious reasons. It is uniquely determined if $l > 0$. In that case bankruptcy occurs if and only if $s < \gamma$. This interpretation of the bankruptcy point fails only if $l = 0$ in which case $W_0 = 0$, γ is immaterial and there is no possibility of bankruptcy. In either case we can use this new representation to simplify the contract problem even further. For any $\gamma \geq 0$ define functions f_γ and g_γ by putting

$$f_\gamma(s, l) = \begin{cases} f(s, l) & \text{if } s \geq \gamma \\ f(s, l) - c_1(s, l) - c_0 & \text{if } s < \gamma \end{cases} \quad (7)$$

and

$$g_\gamma(s, l) = \begin{cases} f(\gamma, l) & \text{if } s \geq \gamma \\ f(s, l) - c_1(s, l) & \text{if } s < \gamma \end{cases} \quad (8)$$

for any $(s, l) \geq 0$. Then the contract problem (3) can be written compactly as

$$\max E[f_\gamma - (1+i)l] \quad (9i)$$

$$\text{s.t. } Eg_\gamma \geq (1+i)(l - W_0) \quad (9ii)$$

$$(l, \gamma) \geq 0. \quad (9iii)$$

This is the problem studied below.

Remark. One question which has been ignored so far is whether the contract we have been calling optimal is individually rational for the entrepreneur. This depends very much on what the alternatives are and, in particular, whether and at what cost the entrepreneur can go bankrupt at date 0. If the question is simply whether he would prefer to undertake the venture rather than not, the appropriate criterion is

$$E[W_1 - Bc_0] \geq 0,$$

assuming as usual that negative consumption is impossible. On the other hand, if he can renege on the firm's debts, keep his own assets because of limited liability and suffers a penalty c_0 then the appropriate criterion is

$$E[W_1 - Bc_0] \geq (1+i)(A_0 - c_0).$$

This inequality will be satisfied whenever $A_0 = 0$ or $R_0 = 0$.

3. THE OPTIMAL INVESTMENT LEVEL

Having characterized the optimal contract as a standard debt contract with maximum equity participation we turn to the central question: how do informational asymmetries affect the level of investment in the presence of bankruptcy/information costs? In particular, do they lead to credit-rationing? In answering this question we make two types of comparison. The first is to compare the second-best-optimal investment level, which results from an incentive-compatible contract, with the first best, which would obtain in a world with complete information. The second is to compare the second-best-optimal investment level with the level resulting from interest-rate-taking behaviour.

3.1. Comparisons with the first best

The first-best level of investment, l^* , will be achieved if both agents directly observe the state of nature. Then

$$l^* \in \arg \max_{l \geq 0} E\{f(l, s) - (1+i)l\}.$$

For any value of $\gamma \geq 0$ define functions $l^*(\cdot)$ and $\underline{l}(\cdot)$ by putting

$$l^*(\gamma) = \arg \max_{l \geq 0} E\{f_\gamma(s, l) - (1+i)l\} \tag{10}$$

$$\underline{l}(\gamma) = \arg \max_{l \geq 0} E\{g_\gamma(s, l) - (1+i)l\}. \tag{11}$$

From Assumptions 1 to 3, $D_2g_\gamma \leq D_2f_\gamma \leq D_2f$ and $l^* < \infty$ so the maximands are bounded above. Since they are also continuous a maximum exists in each case. The maximum is unique because f is strictly concave and c_1 convex in l , so $l^*(\cdot)$ and $\underline{l}(\cdot)$ are well-defined.

The functions $l^*(\cdot)$ and $\underline{l}(\cdot)$ mark the boundaries of the region in which the interests of the entrepreneur and investor conflict. For a given value of γ the investor wants l to move in the direction of $\underline{l}(\gamma)$. For a given value of γ the entrepreneur wants l to move in the direction of $l^*(\gamma)$. An efficient contract must therefore have l lying between $\underline{l}(\gamma)$ and $l^*(\gamma)$ as the next proposition shows.

Proposition 5. *If (l, γ) is an optimal contract then $\underline{l}(\gamma) \leq l \leq l^*(\gamma)$.*

Proof. For any value of γ , $\underline{l}(\gamma) \leq l^*(\gamma)$ because f_γ and g_γ are concave in l and $D_2g_\gamma \leq D_2f_\gamma$. If (l, γ) is optimal and $l > l^*(\gamma)$ a small reduction in l raises both (9i) and (9ii) contradicting optimality. A similar argument shows $l \geq \underline{l}(\gamma)$. ||

Proposition 5 is the essential tool for demonstrating the existence of under-investment, as the next result shows.

Proposition 6. *Let $(\hat{l}, \hat{\gamma})$ be an optimal contract and suppose that $D_2c_1(s, l^*) > 0$ and that $\Pr[s < \hat{\gamma}] > 0$. Then $\hat{l} < l^*$.*

Proof. From the definition of $l^*(\cdot)$ it is obvious that $l^*(0) = l^*$. Since $-c_1$ and f are concave in l , $l^*(\cdot)$ is monotonically non-decreasing. Then $l^*(\hat{\gamma}) = l^*$ implies that $l^*(\gamma) = l^*$ for all $0 \leq \gamma \leq \hat{\gamma}$. This means that $ED_2f_\gamma(s, l^*) = (1+i)$ for all $0 \leq \gamma \leq \hat{\gamma}$. Then $E\{D_2c_1(s, l^*) | s < \hat{\gamma}\} = 0$, a contradiction. ||

The proposition shows that if there is a positive probability of bankruptcy and an increase in investment (at the first-best level) increases the cost of bankruptcy in each state, the second-best optimal investment level is strictly less than the first best. It is obvious that the second best never exceeds the first best since $l^*(\gamma) \leq l^*$ for all $\gamma \geq 0$. The conditions of Proposition 6 are not very strong but they rule out cases where, although the costs of bankruptcy are high, the marginal costs are zero, eg., when $c_1 = 0$. The next proposition is designed to deal with this case. We make assumptions about the distribution of states of nature which are stronger than necessary. The argument can be extended to deal with less-well-behaved measures H but only at the cost of some complications which we seek here to avoid. (See Gale and Hellwig, 1983).

Proposition 7. *Let $(\hat{l}, \hat{\gamma})$ be an optimal contract and suppose the distribution of s on the interval $(\hat{\gamma} - \epsilon, \hat{\gamma} + \epsilon)$ has a continuous density h and $h(\hat{\gamma}) > 0$. If $c_1(\hat{\gamma}, l^*) > 0$ or $c_0 > 0$ then $\hat{l} < l^*$.*

Proof. Suppose to the contrary that $\hat{l} = l^*$. Then define

$$u(l, \gamma) := Ef_\gamma(s, l) - (1+i)(l - W_0)$$

$$v(l, \gamma) := Eg_\gamma(s, l) - (1+i)(l - W_0).$$

Both u and v are C^1 functions in a neighbourhood of $(\hat{l}, \hat{\gamma})$. The constraint qualification of the Kuhn-Tucker Theorem is satisfied since $ED_2g_\gamma(s, \hat{l}) < ED_2f_\gamma(s, \hat{l}) = (1+i)$. Therefore, since $\hat{l} > 0$ and $\hat{\gamma} > 0$ by assumption,

$$u_l(\hat{l}, \hat{\gamma}) + \lambda v_l(\hat{l}, \hat{\gamma}) = 0$$

$$u_\gamma(\hat{l}, \hat{\gamma}) + \lambda v_\gamma(\hat{l}, \hat{\gamma}) = 0.$$

In view of Proposition 6 we must have $ED_2f_\gamma(s, l^*) = ED_2f(s, l^*) = (1+i)$ so $u_l(\hat{l}, \hat{\gamma}) = 0$. But $v_l(\hat{l}, \hat{\gamma}) < 0$, as we have already seen, so $\lambda = 0$. Then $u_\gamma(\hat{l}, \hat{\gamma}) = 0$, contradicting $g_\gamma(s, l) < f(s, l)$. \parallel

Remark. The hypotheses of both propositions refer to the optimal value of γ . It would be better to express these conditions in terms of exogenous variables only. As far as Proposition 6 is concerned we can replace the assumption $\Pr[s < \hat{\gamma}] > 0$ by the assumption

$$l^* - W_0 < (1+i)^{-1} \inf \{f(s, l^*) | s \in \text{supp } H\}. \quad (12)$$

In order to finance the first-best level of investment the entrepreneur must borrow $l^* - W_0$. The maximum amount the venture produces with certainty is the expression on the right-hand side of (12). Then (12) implies that *either* there is positive probability of bankruptcy *or* the zero-profit constraint requires $l < l^*$.

In the case of Proposition 7 it is clear that one could assume instead that $c_1 \gg 0$, H has a continuous positive density everywhere and that (12) holds. But clearly these assumptions are stronger than necessary.

Clearly, the assumption of a positive probability of bankruptcy is not sufficient for $l < l^*$. If $c_1 = c_0 = 0$ then any optimal contract involves $l = l^*$. Likewise the costliness of bankruptcy is not sufficient. If the inequality in (12) were reversed the optimal contract would again require $l = l^*$. In short, one needs both bankruptcy costs and a positive probability of bankruptcy. For the record we state without proof

Proposition 8. *Let (l, γ) be an optimal contract. Then $l = l^*$ if one of the following conditions holds:*

- (i) $D_2c_1 = 0$ and $\gamma = \infty$;
- (ii) $c_1 = c_0 = 0$
- (iii) $f(s, l^*) \geq (1+i)(l^* - W_0)$ for all $s \in \text{supp } H$.

3.2. Comparison with price-taking behaviour

Although it is now generally recognized that utility-taking behaviour is the appropriate representation of competition when markets are incomplete, price-taking behaviour has often been used to analyse competitive capital markets with credit-rationing, especially in the macroeconomics literature. Examples are Hodgman (1962) and Jaffee and Russell (1976). Price-taking, in this context, means that the borrower assumes he can borrow as much as he wants at a given rate of interest. In equilibrium, the rate of interest must be chosen so that the lender's zero-profit constraint is satisfied, but the borrower ignores

this relationship when choosing the “optimal” level of investment. The result of such price-taking or more precisely interest-rate-taking behaviour can be described as a special sort of contract. To simplify the discussion we assume $c_0 = 0$ in what follows. We say that a contract (l, γ) is the result of interest-rate-taking behaviour if three conditions are satisfied. First, the entrepreneur chooses l to maximize his expected utility under the assumption that he can borrow any amount at the interest rate r :

$$l \in \arg \max_{\hat{l} \geq 0} E \max \{f(s, \hat{l}) - (1+r)(\hat{l} - W), 0\}. \tag{13}$$

Second, the investor breaks even on the contract:

$$Eg_\gamma(s, l) = (1+i)(l - W_0) \tag{14}$$

And finally, the notional rate r is the interest rate actually paid by the entrepreneur when he is not bankrupt:

$$(1+r)(l - W_0) = f(\gamma, l) = R_1. \tag{15}$$

It is intuitively plausible and easy to show that under interest-rate-taking behaviour the level of investment is higher than it would be under utility-taking behaviour *with the same notional interest rate*. But it is not at all clear whether the actual level of investment will be higher or lower under interest-rate-taking behaviour than under utility-taking behaviour. What we can show is that the probability of bankruptcy will be higher under interest-rate-taking behaviour. The assumption $W_0 > 0$ is made so that we get a particularly sharp lower bound on l (Proposition 9) and can avail ourselves of a rather neat proof of Proposition 10.

Proposition 9. *Suppose that (l, γ) is the result of interest-rate-taking behaviour and that $\Pr[s < \gamma] > 0$ and $W_0 > 0$. Then $l > l^*(\gamma)$.*

Proof. Clearly $l = 0$ is not optimal. Then letting $g \equiv f - c_1$, (14) and (15) imply

$$\int_{s < \gamma} g(s, l) + \int_{s \geq \gamma} (1+r)(l - W_0) = (1+i)(l - W_0)$$

or

$$\int_{s < \gamma} \frac{g(s, l)}{(l - W_0)} + \int_{s \geq \gamma} (1+r) = (1+i).$$

Then (13) implies

$$\int_{s \geq \gamma} D_2 f(s, l) - (1+r) = 0$$

so

$$\int_{s < \gamma} \frac{g(s, l)}{l - W_0} + \int_{s \geq \gamma} D_2 f(s, l) = (1+i).$$

Since $g(s, \cdot)$ is concave and $l - W_0 < l$, we have

$$\int_{s < \gamma} D_2 g(s, l) + \int_{s \geq \gamma} D_2 f(s, l) < (1+i). \quad \parallel$$

Proposition 10. Let (l, γ) be an optimal contract and $(\hat{l}, \hat{\gamma})$ the result of price-taking behaviour. Then $\gamma \cong \hat{\gamma}$ if the conditions of Proposition 9 are satisfied by $(\hat{l}, \hat{\gamma})$.

Proof. Since $\hat{l} > l^*(\hat{\gamma})$, $(l^*(\hat{\gamma}), \hat{\gamma})$ is feasible. If $\gamma > \hat{\gamma}$ then

$$\begin{aligned} Ef_{\gamma}(s, l) - (1+i)l &< Ef_{\gamma}(s, l^*(\gamma)) - (1+i)l^*(\gamma) \\ &\leq Ef_{\hat{\gamma}}(s, l^*(\hat{\gamma})) - (1+i)l^*(\hat{\gamma}). \quad \parallel \end{aligned}$$

Remark. The assumption $W_0 > 0$ is relaxed in Gale and Hellwig (1983) where it is shown that $\gamma < \hat{\gamma}$ when the following, special, functional forms are used:

$$f(s, l) = s\phi(l) \quad (16)$$

$$c_0 = \alpha, \quad c_1(s, l) = \beta f(s, l), \quad 0 \leq \beta \leq 1, \quad \alpha \geq 0. \quad (17)$$

The comparison of interest-rate-taking and utility-taking behaviour is ambiguous as far as l is concerned. The nearest we can get to a definite result is the following proposition.

Proposition 11. Suppose the conditions of Proposition 10 are satisfied. Then

- (i) $\hat{l} > l$ if $Eg_{\hat{\gamma}}(s, l) > Eg_{\gamma}(s, l)$
- (ii) $l < \hat{l}$ if $Eg_{\gamma}(s, \hat{l}) > Eg_{\hat{\gamma}}(s, \hat{l})$.

Proof. The zero-profit constraint implies

$$Eg_{\gamma}(s, l) - (1+i)l = Eg_{\hat{\gamma}}(s, \hat{l}) - (1+i)\hat{l}$$

so (i) implies

$$E\{g_{\gamma}(s, l) - g_{\hat{\gamma}}(s, \hat{l}) - (1+i)(l - \hat{l})\} > 0.$$

Concavity implies

$$E\{D_2g_{\gamma}(s, \hat{l}) - (1+i)\}(l - \hat{l}) > 0.$$

Since $\hat{l} > l^*(\hat{\gamma}) \cong l(\hat{\gamma})$ this implies that $l - \hat{l} < 0$. A similar argument works in the second case. \parallel

Remark. In Gale and Hellwig (1983) this result is extended to the case where $W_0 < 0$ for the functional forms (16) and (17). Proposition 11 implies that interest-rate-taking behaviour always leads to higher investment if $g_{\gamma}(s, l)$ is increasing in γ , as would be the case in (17) if $\alpha > 0$ and $\beta = 0$.

4. RISK-AVERSION

In this section we consider briefly the effects of risk-aversion on the optimal contract. It turns out that the form of the contract is altered but much of the remaining analysis is unaffected. The discussion below is kept deliberately informal. Without risk-neutrality the analysis becomes much more difficult and we do not have space to deal with technicalities here. We continue to assume $c_0 = 0$ to simplify the analysis. Non-pecuniary penalties are obviously hard to incorporate (and also rather unnecessary) when we have risk aversion.

We maintain the assumption that the investor is risk-neutral. Without this assumption we cannot analyse the optimal contract between an investor and an entrepreneur indepen-

dently of the other contracts the investor may be involved in. However, risk-neutrality is not an unreasonable assumption to make in the case of investors since it can be justified as a consequence of risk-pooling. It makes less sense in the case of entrepreneurs and indeed is merely a “simplifying” assumption which ought to be relaxed if possible.

Suppose then that the entrepreneur has a strictly concave Neumann–Morgenstern utility $U(W_1)$ with the usual mathematical properties. The derivation of the constraints in the optimal contract problem (3) is unaffected by the change in the objective function so we can see immediately that the optimal contract problem is:

$$\max EU(W_1) \tag{16i}$$

$$\text{s.t. } EC_1 \cong (1+i)(l+R_0-C_0); \tag{16ii}$$

$$W_1 + C_1 \cong f - Bc_1 + (1+i)(A_0 - C_0); \tag{16iii}$$

$$l \cong 0, \quad 0 \leq C_0 \leq A_0, \quad W_1 \cong 0; \tag{16iv}$$

$$\text{for some } R_1, B(R - C_1) \cong Bc_1 \text{ and } (1 - B)(C_1 - R) = 0. \tag{16v}$$

We now deal with each of the results in Sections 2 and 3 in turn.

Proposition 2 continues to hold. Every optimal contract in the sense of (16) is weakly dominated by a contract with MEP. The proof is the same as before.

Proposition 3 does not hold. Trivially, $C_1 = R_1$ when the firm is not bankrupt (16v) but in the event of bankruptcy we typically have $W_1 > 0$ and hence $C_1 < f - c_1$. For example $W_1 > 0$ in almost every state if $U'(0) = \infty$. What we do have still is the condition that

$$B(W_1 - D) = 0 \text{ for some constant } D. \tag{17}$$

Because the entrepreneur is risk-averse and the investor risk-neutral the entrepreneur gets full insurance in those states which are observed by both agents.

Suppose that the states of nature are continuously distributed. This assumption is maintained throughout the rest of the section. Then there exists an optimal contract in which B has the form $B = \chi_{\{s < \gamma\}}$. To see this choose γ so that the probability of bankruptcy is the same as in the given optimal contract. Assumption 2 implies that $f - Bc_1$ has not fallen. The probability that $W_1 = D$ is the same and $(1 - B)(f - R_1)$ weakly stochastically dominates the previous distribution. In the risk-neutral case, any optimal contract was weakly dominated by a SDC (l, R_1) . Here it is weakly dominated by a contract which can be parameterized by (l, γ, R_1, D) . Note that the bankruptcy point γ need not satisfy $R_1 = f(\gamma, l)$.

Proposition 4. We showed above that an optimal contract was weakly dominated by one in which $B = \chi_{\{s < \gamma\}}$. This is a necessary condition for optimality if there is a positive saving in bankruptcy costs, e.g. if $c_1(s, l)$ is strictly increasing in s and the probability of bankruptcy is positive.

Proposition 5 continues to hold. An optimal contract of the form (l, γ, R, D) solves the problem

$$\max \int_{s < \gamma} U(D) + \int_{s \cong \gamma} U(f - R_1) \tag{18i}$$

$$\text{s.t. } \int_{s < \gamma} f - c_1 - D + \int_{s \cong \gamma} R_1 = (1+i)(l - W_0); \tag{18ii}$$

$$l \geq 0, \quad \gamma \geq 0, \quad D \geq 0 \quad \text{and} \quad f(s, l) - R_1 \geq 0 \quad (18ii)$$

for $s \geq \gamma$;

$$f(\gamma, l) - R_1 \leq D. \quad (18iv)$$

(18i)-(18iii) are simple translations of the corresponding constraints in (16). (18iv) is the remaining incentive-compatibility constraint. If $l < \underline{l}(\gamma)$ then a small increase makes both agents better off if R_1 is adjusted to keep $f(\gamma, l) - R_1$ constant. This also ensures that (18iv) is satisfied. If $l > l^*(\gamma)$ then a small reduction in l , adjusting R_1 to keep $f(\gamma, l) - R_1$ constant, certainly makes the investor better off. If we adjust D so that (18ii) continues to hold exactly then, assuming $D > 0$ and using the fact that

$$\int_{s < \gamma} U'(D) = \int_{s \geq \gamma} U'(f - R_1),$$

we find after a simple calculation that

$$dU = \int_s U'(D)(D_2 f_\gamma(s, l) - (1+i)) dl > 0.$$

Propositions 6 and 7 continue to hold since they depend only on the fact that $\underline{l}(\gamma) \leq l \leq l^*(\gamma)$ and the properties of $\underline{l}(\cdot)$ and $l^*(\cdot)$.

Proposition 8 clearly continues to hold independently of the entrepreneur's objective function.

Propositions 9 and 10. The analysis of interest-rate-taking behaviour is much more complicated under risk-aversion. We do not yet know whether analogues to these propositions exist in the risk-averse case.

5. MONOTONICITY

In Gale (1983a and 1983b) it was shown that liquidity constraints reduce a firm's demand for labour in much the same way as quantity constraints. Furthermore, these constraints give rise to "Keynesian" phenomena just as quantity constraints do. For example, fiscal policy has a multiplier effect on employment and output of the sort predicted by simple Keynesian models. The results in the preceding pages show that effective demand failures occur even when bankruptcy is allowed (in Gale, 1983a and 1983b there is a strict "no bankruptcy" condition) and agents write general, optimal contracts. The underinvestment result ($l < l^*$) can also be interpreted as an underemployment result and generalizes the liquidity-constraint analysis of Gale (1983a and 1983b). An interesting question is whether a reduction in liquidity aggravates the under-investment problem. That is, does a fall in W_0 reduce l ? The answer, perhaps surprisingly, is not necessarily.

A simply example will make this clear. Suppose that $c_1 = 0$ and $c_0 > 0$. We assume that both agents are risk-neutral. If W_0 is sufficiently large then $l = l^*$ (Proposition 10). On the other hand, if W_0 is so small⁷ that $\Pr[s \leq \gamma] = 1$ then $\underline{l}(\gamma) = l^*$. More precisely, when $c_1 = 0$, Eg_γ is maximized with respect to γ at $\hat{\gamma} = \sup\{s | H(s) < 1\}$. When

$$W_0 = -(1+i)^{-1}(Ef(s, l^*) - (1+i)l^*)$$

the only contracts satisfying the zero-profit constraint have the form (l^*, γ) with $\gamma \geq \hat{\gamma}$. Thus $l = l^*$ for high and low values of W_0 . However, Proposition 9 gives conditions under which $l < l^*$ for some intermediate values of W_0 . Thus, the relationship between l and W_0 is at best U-shaped.

Proposition 13. *Suppose that $c_1 \equiv 0$, $c_0 > 0$ and H has a continuous, strictly positive density on $[0, \hat{\gamma}]$. Then the optimal value of l cannot be monotonic in W_0 .*

A complete analysis of the relationship between l and W_0 , which amounts to the complete analysis of the structure of the efficient set of contracts, is extremely difficult. In Gale and Hellwig (1983) we have carried out an exhaustive study of the structure of the efficient set for the class of models characterized by (16) and (17). The example given above is by no means untypical.

First version received October 1983; final version accepted April 1985 (Eds.).

We thank Helmut Bester, Oliver Hart, Laurenz Kohlleppe, John Moore, Kevin Roberts and two anonymous referees for their comments on earlier drafts. Research support from the Deutsche Forschungsgemeinschaft and from the ICERD is gratefully acknowledged.

NOTES

1. In contrast we have little to say about the stronger form of credit-rationing as contract-rationing that Stiglitz and Weiss (1981) recently derived from assumptions of asymmetric information about project quality and project choice. We confirm their basic propositions that increases in contractual interest payments will not necessarily raise expected returns on lending. However, in our analysis this observation does not entail credit-rationing because, unlike Stiglitz and Weiss, we assume investments are perfectly divisible.

2. A referee has suggested that it is this assumption and not the divisibility of investment that constitutes the major difference between us and Stiglitz-Weiss. (See footnote 1 above). For example, Stiglitz-Weiss might be seen as an extreme case of our model with strictly increasing returns up to some maximum investment level. Now this is a pathological case; without the kink there would at least be locally diminishing returns at the first-best level. In any case, the argument ignores the other important differences, specifically informational assumptions and observation costs.

3. With perfect risk-pooling investors are able to absorb arbitrarily large losses on any one venture (though in practice losses are bounded below). This ability constitutes an important asymmetry between investors and entrepreneurs. Because entrepreneurs invest in only one venture their losses on that venture are restricted to their initial capital.

4. The notation $D_{i,j}f$ indicates the partial derivative of f with respect to the i -th argument. Interpret $D_{i,j}f$ similarly.

5. The set of states of nature is taken to be \mathbb{R}_+ . All random variables are measurable and real-valued functions defined on the measure space $(\mathbb{R}_+, \mathcal{B}(\mathbb{R}_+))$ where $\mathcal{B}(\mathbb{R}_+)$ denotes the Borel sets of \mathbb{R}_+ . Expected values of random variables are integrals with respect to H .

6. Inequalities and equations involving random variables are naturally assumed to hold almost everywhere, i.e., with probability 1.

7. The question arises whether individual rationality is always satisfied as we vary W_0 . Let us put $R_0 = \min\{W_0, 0\}$ and $A_0 = \max\{W_0, 0\}$. If the entrepreneur suffers a penalty $c_0 > 0$ for renegeing at date 0 he will strictly prefer any admissible contract to bankruptcy at date 0. (See the remark at the end of Section 2. Even if he goes bankrupt with probability 1 at date 1 the penalty is delayed and therefore discounted.)

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