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SMOOTH PREFERENCES: A CORRIGENDUM

BY GERARD DEBREU¹

MARCEL K. RICHTER recently informed me of conversations that he had with Andreu Mas-Colell and Leonard Shapiro about one aspect of the problem of local integrability in the theory of differentiable preferences, and mentioned the following example given, in a different context, by Alan Weinstein in a mathematics course at the University of California, Berkeley.

Using the notation of [1], consider in a neighborhood of 0 in R^2 the problem of integrability of the unit vector field g defined by:

$$g_1(x, y) = \frac{y^2}{\sqrt{1 + y^4}}, \quad g_2(x, y) = \frac{1}{\sqrt{1 + y^4}}, \quad \text{for } y \geq 0,$$

and

$$g_1(x, y) = 0, \quad g_2(x, y) = 1, \quad \text{for } y \leq 0.$$

g is C^1 (and, of course, locally integrable), but not C^2 . Associated with it is a utility function u defined by $u(x, y) = y/(1 - xy)$, for $y \geq 0$, and $u(x, y) = y$, for $y \leq 0$.

u is C^1 and $Du(0) \neq 0$, but, as A. Mas-Colell noted, there is no utility function v associated with the vector field g that is C^2 , and for which $Dv(0) \neq 0$. Thus, the assertion I made on this point in [1, p. 606, lines 26–28] is incorrect. On line 28, “ C^2 ” must be replaced by “ C^1 ”. By the same token, in [1, p. 610, lines 40–44] (ii) does not imply (iii).

However, the assertion of the equivalence of (i) and (iii) is valid (for a proof see [4, footnote 1]), and (iii) trivially implies (ii).

It must also be observed that C^1 -differentiability of the demand function [1, pp. 612–613] can be obtained even though there is no C^2 utility function without critical point. Assume that g is C^1 and locally integrable. Then every indifference hypersurface is C^1 and has a C^1 normal, which readily implies that it is actually a C^2 -hypersurface. Assume, in addition, that g is positive; that it satisfies the condition of convexity and nonzero Gaussian curvature “for every x in $P = \text{Int } R^l_+$, for every y in R^l , $y \neq 0$, such that $y \cdot g(x) = 0$, one has $y \cdot J(g, x)y < 0$ ” where $J(g, x)$ is the Jacobian of g at x ; and that for every x in P , the closure relative to R^l of the indifference hypersurface of x is contained in P . Let $S = \{p \in P \mid \|p\| = 1\}$ and $L = \text{Int } R_+$. The preceding conditions imply that there is a well defined demand function from $S \times L$ to P . Now consider the C^1 function $\phi: P \rightarrow P$ defined by $\phi(x) = (g_1(x), \dots, g_{l-1}(x), x \cdot g(x))$. The Jacobian determinant of ϕ at x can directly be shown to equal

$$\begin{vmatrix} \partial_j g_i \\ g_j \end{vmatrix} \quad (i = 1, \dots, l - 1; j = 1, \dots, l)$$

which, in turn, equals cg_l , where c denotes Gaussian curvature. Since cg_l is different from zero, the demand function ϕ^{-1} is C^1 .

Finally, it may be worth noting that all the assertions made in [1] would be correct if it were written in the context of C^∞ functions and hypersurfaces. A justification for working in that context is provided by the approximation results of [2 and 3].

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