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SMOOTH PREFERENCES: A CORRIGENDUM

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MARCEL K. RICHTER recently informed me of conversations that he had with Andreu Mas-Colell and Leonard Shapiro about one aspect of the problem of local integrability in the theory of differentiable preferences, and mentioned the following example given, in a different context, by Alan Weinstein in a mathematics course at the University of California, Berkeley.

Using the notation of [**1**], consider in a neighborhood of 0 in R^2 the problem of integrability of the unit vector field g defined by:

$$g_1(x, y) = \frac{y^2}{\sqrt{1 + y^4}}, \qquad g_2(x, y) = \frac{1}{\sqrt{1 + y^4}}, \qquad \text{for } y \geqslant 0,$$

and

$$g_1(x, y) = 0, \qquad g_2(x, y) = 1, \qquad \text{for } y \leqslant 0.$$

g is C^1 (and, of course, locally integrable), but not C^2. Associated with it is a utility function u defined by $u(x, y) = y/(1 - xy)$, for $y \geqslant 0$, and $u(x, y) = y$, for $y \leqslant 0$.

u is C^1 and $Du(0) \neq 0$, but, as A. Mas-Colell noted, there is no utility function v associated with the vector field g that is C^2, and for which $Dv(0) \neq 0$. Thus, the assertion I made on this point in [**1**, p. 606, lines 26–28] is incorrect. On line 28, "C^2" must be replaced by "C^1". By the same token, in [**1**, p. 610, lines 40–44] (ii) does not imply (iii).

However, the assertion of the equivalence of (i) and (iii) is valid (for a proof see [**4**, footnote 1]), and (iii) trivially implies (ii).

It must also be observed that C^1-differentiability of the demand function [**1**, pp. 612–613] can be obtained even though there is no C^2 utility function without critical point. Assume that g is C^1 and locally integrable. Then every indifference hypersurface is C^1 and has a C^1 normal, which readily implies that it is actually a C^2-hypersurface. Assume, in addition, that g is positive; that it satisfies the condition of convexity and nonzero Gaussian curvature "for every x in $P = \text{Int } R^l_+$, for every y in R^l, $y \neq 0$, such that $y \cdot g(x) = 0$, one has $y \cdot J(g, x)y < 0$" where $J(g, x)$ is the Jacobian of g at x; and that for every x in P, the closure relative to R^l of the indifference hypersurface of x is contained in P. Let $S = \{p \in P | \ \|p\| = 1\}$ and $L = \text{Int } R_+$. The preceding conditions imply that there is a well defined demand function from $S \times L$ to P. Now consider the C^1 function $\phi: P \to P$ defined by $\phi(x) = (g_1(x), \ldots, g_{l-1}(x), x \cdot g(x))$. The Jacobian determinant of ϕ at x can directly be shown to equal

$$\begin{vmatrix} \partial_j g_i \\ g_j \end{vmatrix} \qquad (i = 1, \ldots, l-1; j = 1, \ldots, l)$$

which, in turn, equals cg_l, where c denotes Gaussian curvature. Since cg_l is different from zero, the demand function ϕ^{-1} is C^1.

Finally, it may be worth noting that all the assertions made in [**1**] would be correct if it were written in the context of C^∞ functions and hypersurfaces. A justification for working in that context is provided by the approximation results of [**2** and **3**].

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