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Growth and welfare effects of monetary policy with endogenous fertility

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ABSTRACT

This paper investigates the economic growth and social welfare implications of monetary policy in an endogenous growth model with endogenous fertility. We show that, in the money-in-the-utility-function framework, endogenous fertility governs the validity of money superneutrality, the transitional dynamics of an anticipated monetary policy, and the optimal monetary policy. Along a balanced growth path, monetary growth increases fertility and reduces the economic growth rate if consumption and real balances are complements or are independent. However, monetary expansion may decrease fertility and increase economic growth if consumption and real balances are substitutes. Generally speaking, the superneutrality of money does not hold in the presence of endogenous fertility. More importantly, with endogenous fertility, the Friedman (1969) rule is no longer a welfare-maximizing monetary policy. We also show that an anticipated inflation induces the transitional dynamics of fertility and the economic growth rate even though the intertemporal elasticity of substitution equals unity. This differs from the conventional notion in the existing literature.

1. Introduction

Drastic transitions in the demographic structure of developed and developing countries over the past several decades have stimulated considerable interest in how population affects economies. Economists have paid much attention to the role of fertility in economic growth issues. The literature includes studies on the existence of multiple equilibria or balanced growth paths in economies (e.g., Becker et al., 1990; Palivos, 1995), on optimal population growth and optimal economic growth rates (e.g., Palivos and Yip, 1993), on the effects of fertility shocks (e.g., Wang et al., 1994), on income inequality and economic growth (e.g., De la Croix and Doepke, 2003), on the fertility externality and the optimal fiscal policy (e.g., Van Groezen and Meijdam, 2008), and on social status and economic growth (e.g., Tournemaine, 2008). Obviously, these studies mentioned above restrict their analysis to real-side issues. To the best of our knowledge, an exception is Yakita (2006) who examines the effect of an increase in life expectancy on the portfolio choices of individuals and, thereby, on economic growth and inflation in a monetary model of endogenous growth populated by overlapping generations. Our focus differs from his. To highlight the importance of the family decision regarding fertility in a monetary framework, this paper aims to explore the positive and normative implications of monetary policy in an endogenous growth model with endogenous fertility.

Analyzing the relationship between monetary policy and economic growth is a central issue in macroeconomics. Due to the excellent contributions of Romer (1986) and Lucas (1988), many economists focus on endogenous growth theory and are

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interested in determining whether or not monetary policy can influence long-term economic growth. Previous studies consider the importance of how money enters the economy to examine the effect of inflation on the balanced growth rate (e.g., Marquis and Reffett, 1991; Wang and Yip, 1992; Van der Ploeg and Alogoskoufis, 1994; Mino and Shibata, 1995; Chang and Lai, 2000; Chang, 2002; Jha et al., 2002). De Gregorio (1993), Zhang (1996), and Mino (1997) focus on the role of a real factor, such as the endogenous labor decision, which the real business cycle literature emphasizes. Chang et al. (2000) concentrate on the effects of pursuing a wealth-motivated status; and agents engage in the accumulation of wealth, lending impetus to the long-term growth rate. Recently, Itaya and Mino (2007) have demonstrated that preference structures and production technology are important factors in assessing the effects of monetary policy in a growth model characterized by increasing returns and endogenous labor choices.

Empirical studies also reveal an interest in fertility behaviors. They examine the factors affecting fertility, such as the wage, per capita output, taxation, and transfer programs (e.g., Willis, 1973; Hotz and Miller, 1988; Whittington et al., 1990; Zhang et al., 1994; Milligan, 2005; Björklund, 2006). Do any other factors, such as financial conditions, influence fertility decisions? This phenomenon may occur in some developing countries, such as Central Eastern European countries. By using Ukrainian data from 1996 to 2005, Maksymenko (2009) showed that there exists a relationship between financial wealth and fertility; financial conditions, such as money holdings, account for the changes in fertility in Eastern Europe. This finding implies that if fertility is linked to money holdings, the government can influence fertility via monetary policy.

To respond to this empirical finding, this paper will uncover the long-term effects of an increase in the money growth rate on fertility, economic growth and social welfare in a macro-model with endogenous fertility by adopting both analytical and numerical approaches. To the end, we incorporate the role of endogenous fertility into the Barro (1990) and Rebelo (1991)-type endogenous growth framework with money in the utility function. Based on this amended model, we obtain several novel and interesting results. In a benchmark model in which consumption and real balances are independent, we find that the superneutrality of money is not valid. Intuitively, monetary expansion raises the inflation rate and thus increases the cost of real money holdings, which in turn decreases real money balances. Given an endogenous fertility rate, this gives rise to the co-called wealth-narrowing effect (a large population dilutes the stock of wealth per person) which reduces the households' cost of raising children. As a result, the fertility rate increases and, accordingly, the balanced-growth rate decreases. In the related literature, Van der Ploeg and Alogoskoufis (1994) introduce the uncertain-lifetimes approach of Blanchard (1985) with the money-in-the-utility-function setting of Sidrauski (1967) to Lucas's (1988) model and show that an increase in the rate of monetary growth through open-market operations raises long-run growth. Mino and Shibata (1995) construct an overlapping-generations model with a simple convex production technology and Sidrauski's (1967) money-in-the-utility-function approach. They conclude that an increase in the money growth rate has a positive impact on the long-run growth rate through the intergenerational redistribution effects. Based on Sidrauski's (1967) framework, Chang and Lai (2000) develop a monetary endogenous growth model, in which the AK production technology is adopted and money enters the utility function, and refer to the superneutrality of money. Our result obviously contrasts with those of previous studies.

In an extended model with interdependence between consumption and real balances, in addition to the wealth-narrowing effect, the preference effect also plays a role in terms of influencing the effects of monetary policy on fertility and growth. A particular emphasis is that if consumption and real balances are substitutes, the preference gives rise to an opposite impact, perhaps resulting in a lower fertility rate and a higher growth rate. Besides, we also find that even if the intertemporal elasticity of substitution equals unity, fertility behavior crucially induces the transitional dynamics of the economy in response to an *anticipated* change in the money growth rate. This outcome differs from that of Chang and Lai (2000), who show that when the intertemporal elasticity of substitution equals unity, the economic growth rate does not react to an *anticipated* rise in the money growth rate.

The traditional welfare analysis indicates that the optimal monetary policy follows the Friedman (1969) rule in the sense that, to achieve the social optimum, the government should eliminate the wedge between the private and social costs of money holding by setting a zero nominal interest rate, corresponding to a zero inflation tax. However, our normative analysis shows that, in the presence of endogenous fertility, the Friedman rule is no longer optimal and crucially depends on households' fertility behaviors.[1] Due to the wealth-narrowing effect stemming from the variable fertility rates, the government should tax the holding of money by setting a positive nominal interest rate. Interestingly, either a lower cost of child-raising or a stronger fertility preference amplifies the wealth-narrowing effect, leading to a higher level of the nominal interest rate.

Finally, we emphasize that in some respects endogenous fertility and endogenous leisure share similar characteristics, but our study provides very different implications of endogenous fertility for monetary policy from the analysis with an endogenous labor-leisure choice, e.g., Brock (1974). His model predicts that monetary neutrality holds if leisure and real balances are separable in the utility function. By contrast, this paper predicts that even if fertility and money are separable in the utility function, monetary policy can still affect economic growth. More importantly, due to the wealth-narrowing effect stemming from endogenous fertility, we can provide a distinctive and novel welfare implication for monetary policy, compared to the study with an endogenous labor-leisure choice.

The remainder of this paper is organized as follows. Sections 2 and 3 build a monetary endogenous growth model with endogenous fertility and, accordingly, examine the effects of an increase in the money growth rate on fertility, growth, and welfare. In addition, transition analysis is also conducted. Section 4 further provides extended analyses and performs a simple numerical exercise. Section 5 concludes.

[1] Van der Ploeg and Alogoskoufis (1994), Mino and Shibata (1995), and Chang and Lai (2000) do not engage in welfare analysis.

2. The model

We incorporate an endogenously-determined fertility rate into a standard endogenous growth model. Consider an economy consisting of a government and a family that is inhabited by a continuum of identical agents. By following Sidrauski (1967), money enters the model through providing the flow of services that enhances the utility directly. In addition to consumption and money, the representative agent also derives utility from child-raising, since the agent enjoys the happiness of child-raising based on an emotional factor, such that the fertility rate enters the utility function (e.g., Wang et al., 1994; Palivos, 1995; Yip and Zhang, 1997). Thus, in line with Brock (1974), Asako (1983), Blackburn and Cipriani (1998), De la Croix and Doepke (2003), Barro and Sala-i-Martin (2004), and Yakita (2006), the lifetime utility function U of the agent is given by[2]:

$$U = \int_0^\infty [\ln c + \alpha \ln m + \beta \ln n] e^{-\rho t} dt, \quad \alpha, \beta > 0 \tag{1}$$

where c is per capita consumption, m ($\equiv M/PN$) is per capita real money balances, M is nominal money holdings, P is the price level, N is the population size or the family size, n is the rate of fertility, ρ is a constant rate of time preference, and β (α) is a parameter capturing the desire for fertility (money) relative to consumption. The utility function satisfies the Inada conditions.

The agent's income is derived from output and government transfers. This income is available for expenditures on consumption, child-rearing costs, inflation tax payments, and the accumulation of wealth. The flow budget constraint faced by the representative agent can be expressed as:

$$\dot{m} + \dot{k} = y + \tau - (\pi + n)m - nk - c - qn, \tag{2}$$

where an overdot is the time derivative, k is the per capita capital stock, $y \equiv Ak$ is the output, A is the technology level, $\tau > 0$ (< 0) is per capita real transfers (lump-sum taxes), $\pi \equiv \dot{P}/P$ is the inflation rate, and q is the per capita cost of child-raising. By following Blackburn and Cipriani (1998) and Barro and Sala-i-Martin (2004), we assume that the costs of child-raising increase with capital and, specifically, take the following form:

$$q = \bar{q}k, \tag{3}$$

where $\bar{q} > 0$ is a constant parameter. This feature displayed in (3) arises from the fact that the child-related activity is time-intensive and the corresponding opportunity cost rises with capital accumulation and technological progress. Thus, without loss of generality and for simplicity, the per capita cost of child-rearing can be assumed to be increasing in physical capital.

Assume that λ is the co-state variable associated with the flow budget constraint (2), which is the shadow price of real wealth. We then obtain the first-order conditions with respect to c, n, m, and k as:

$$1/c = \lambda, \tag{4a}$$
$$\beta/n = \lambda[(m + k) + \bar{q}k], \tag{4b}$$
$$(\alpha/m) - \lambda(\pi + n) = -\dot{\lambda} + \lambda\rho, \tag{4c}$$
$$\lambda(A - n - \bar{q}n) = -\dot{\lambda} + \lambda\rho, \tag{4d}$$

together with (2) and (3), and the transversality conditions of k and m:

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = 0 \quad \text{and} \quad \lim_{t\to\infty} \lambda m e^{-\rho t} = 0. \tag{5}$$

Eq. (4a) is the first-order condition for consumption, implying that the marginal utility of consumption equals the marginal cost of consumption, where the latter is equal to the shadow value of wealth. The optimal condition for fertility (4b) then equalizes its marginal benefits β/n and costs $\lambda[(m + k) + \bar{q}k]$. Notice that a higher fertility rate generates two different costs in the economy as we account for endogenous fertility. The first one refers to the opportunity costs of raising children in terms of foregone capital $\bar{q}k$. The second one, given by ($m + k$), results in the wealth-narrowing effect in the sense that a large population dilutes the stock of wealth per person. Eq. (4c) is the optimal holdings of real money balances according to the difference between the marginal utility of real money balances and the sum of the inflation rate and its child-related costs, adjusted by the shadow value of wealth. Finally, (4d) determines the optimality condition governing capital accumulation by evaluating the net marginal productivity of capital.

The government finances its real transfers by levying seigniorage so as to maintain a balanced budget:

$$\tau = \mu m, \tag{6}$$

where $\mu \equiv \dot{M}/M$ is the constant growth rate of the nominal money stock, which is controlled by the government. Moreover, when the money growth rate increases, the government can rely on endogenously- adjusted real transfers to maintain a balanced budget. By definition, the law of motion for real money balances is given by:

$$\dot{m}/m = \mu - \pi - n. \tag{7}$$

[2] For simplicity, we omit the time subscript throughout this paper.

Combining the government budget constraint (6), the law of motion for real money balances (7), and the agent's budget constraint (2) with (3) yields the economy-wide resource constraint:

$$\dot{k} = Ak - (1+\bar{q})nk - c. \tag{8}$$

It states that the capital accumulation of the economy equals the difference between output and the sum of child-related costs and consumption.

We are in a position to embody the macroeconomic equilibrium of the economy. Following Faig (1995), Mino and Shibata (1995), and Barro and Sala-i-Martin (2004), we define the following transformed variables:

$$x \equiv c/k \quad \text{and} \quad z \equiv m/k, \tag{9}$$

where c, m, and k all grow at the same rate $\tilde{g}$ along the balanced growth path.

By using (4a) and (4b), we solve the fertility decision in terms of the transformed variables x and z as:

$$n = n(x, z, \beta, \bar{q}) = \beta x/(1+\bar{q}+z), \tag{10a}$$

where $n_x = \beta/\Sigma > 0$, $n_z = -\beta x/\Sigma^2 < 0$, $n_\beta = x/\Sigma > 0$, $n_{\bar{q}} = -\beta x/\Sigma^2 < 0$, and $\Sigma = (1+\bar{q}+z) > 0$. As is evident, either a stronger preference on the happiness of child-raising β or fewer opportunity costs of child-raising $\bar{q}$ will have a positive effect on fertility n. To describe the rationale for the effects of x and z, we rewrite (10a) as follows:

$$\beta n^{-1} = (1+\bar{q}+z)x^{-1}, \tag{10b}$$

where the LHS is the marginal utility of fertility and the RHS is the marginal cost of fertility in terms of the transformed variables x and z. Since a higher level of consumption is associated with a lower marginal utility, an increase in the consumption-capital ratio x leads the households to have more concern for their happiness from child-raising and less concern for their consumption-induced utility. This enhances fertility n. Moreover, an increase in the real balances-consumption ratio zx^{-1} reinforces the negative wealth-narrowing effect on fertility, as noted previously. Given that the cost of child-raising grows, the fertility rate is thus depressed. In addition, with (4a) and (9), we can use (4c) and (4d) to obtain the inflation rate:

$$\pi = (\alpha x/z) - A + \bar{q}n, \tag{11}$$

where the term $\alpha x/z$ represents the nominal interest rate with endogenous fertility.[3]

By combining (7) and (11), the money market equilibrium is given by:

$$\dot{m}/m = \mu - (\alpha x/z) + A - \bar{q}n - n. \tag{12}$$

By using (4a) and (4d), we obtain the modified Keynes–Ramsey rule as follows:

$$\dot{c}/c = A - n - \bar{q}n - \rho. \tag{13}$$

With (9) and (10a) and from (8), (12), and (13), the dynamic behavior of the economy in terms of the transformed variables x and z can be expressed as:

$$\dot{x}/x \equiv \dot{c}/c - \dot{k}/k = x - \rho, \tag{14a}$$

$$\dot{z}/z \equiv \dot{m}/m - \dot{k}/k = \mu - (\alpha x/z) + x. \tag{14b}$$

Eqs. (10a), (13), (14a), and (14b) characterize the macro-economy. In what follows, we then utilize these equations to examine the effects of monetary policy on fertility, economic growth, transitional dynamics, and social welfare.

3. The effects of an increase in the money growth rate

This section will discuss both the long-run steady-state effects and the short-run transitional dynamics of increasing the money growth rate on fertility and the balanced growth rate.

3.1. Steady-state effects of monetary growth

Before we perform a steady-state growth equilibrium analysis, we first examine the local stability to ensure that a unique perfect-foresight path leads to the steady-state growth equilibrium. By letting μ_0 be the initial money growth rate, linearizing (14a) and (14b) around the steady state yields:

[3] Eq. (11) can be rewritten as:

$$\alpha x/z = (A - \bar{q}n) + \pi,$$

where the first term on the RHS, $A - \bar{q}n$, denotes the net marginal product of capital and thus the real interest rate, and the second term on the RHS, π, denotes the inflation rate. Accordingly, the sum of these two terms, $\alpha x/z$, represents the nominal interest rate.

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix}=\begin{bmatrix}\tilde{x} & 0\\ \tilde{z}[-(\alpha/\tilde{z})+1] & \tilde{z}(\alpha\tilde{x}/\tilde{z}^2)\end{bmatrix}\begin{bmatrix}x-\tilde{x}\\ z-\tilde{z}\end{bmatrix}+\begin{bmatrix}0\\ \tilde{z}\end{bmatrix}(\mu-\mu_0), \tag{15}$$

where $\tilde{x}$ and $\tilde{z}$ are the steady-state levels of x and z, respectively.

Letting υ_1 and υ_2 be the characteristic roots of (15), we have the following relationships:

$$\upsilon_1+\upsilon_2=(\alpha\tilde{x}/\tilde{z})+\tilde{x}>0, \tag{16a}$$

$$\upsilon_1\upsilon_2\equiv\Delta_1=\tilde{x}(\alpha\tilde{x}/\tilde{z})>0. \tag{16b}$$

Eqs. (16a) and (16b) indicate that the two roots are positive; to be more specific, they are $\upsilon_1=\tilde{x}>0$ and $\upsilon_2=\alpha\tilde{x}/\tilde{z}>0$. As claimed in the literature on dynamic rational expectations models (e.g., Buiter, 1984; Turnovsky, 1995), the economy has a unique perfect-foresight equilibrium if the number of unstable roots is equal to the number of jump variables. Because the economy with two positive unstable roots has two jump variables, x and z, there exists a unique, stable perfect-foresight equilibrium path.

At the steady-state growth equilibrium, the economy is characterized by $\dot{x}=\dot{z}=0$, $x=\tilde{x}$, and $z=\tilde{z}$. From (10a) and (15), we obtain[4]:

$$\partial\tilde{n}/\partial\mu=-\tilde{x}\tilde{z}n_z/\Delta_1>0, \tag{17a}$$

which states that a higher money growth rate affects fertility positively. Furthermore, (17a) and (13) allow us to obtain:

$$\partial\tilde{g}/\partial\mu=-(1+\bar{q})(\partial\tilde{n}/\partial\mu)=\tilde{x}\tilde{z}\cdot(1+\bar{q})n_z/\Delta_1<0, \tag{17b}$$

which states that an increase in the money growth rate decreases the balanced growth rate.

The rationale for these results of (17a) and (17b) is as follows. Monetary expansion raises the inflation rate and thus increases the cost of real money holdings, which in turn decreases real money balances. As shown in (4b), lower real balances m imply that the wealth-narrowing effect of population growth becomes less pronounced, reducing the cost of child-raising. As a result, the fertility rate increases in response to the monetary expansion. Given that the child-related activities have an opportunity cost in terms of foregone capital, in response to a monetary expansion, the returns on capital thus decline and, accordingly, the balanced-growth rate falls. We summarize these results in the following proposition:

Proposition 1. *When endogenous fertility is present, an increase in the money growth rate stimulates fertility and depresses the long-term growth rate.*

The results of Proposition 1 differ from the findings of Van der Ploeg and Alogoskoufis (1994), Mino and Shibata (1995), and Chang and Lai (2000) in the absence of endogenous fertility. Van der Ploeg and Alogoskoufis (1994) extend the Lucas (1988) model to the uncertain-lifetimes approach of Blanchard (1985) with money in the utility function. They conclude that a rise in monetary growth through open-market operations increases long-run growth. Mino and Shibata (1995) employ an overlapping-generations model with money in the utility function and show that changes in the money growth rate have intergenerational redistribution effects that positively affect economic growth. Chang and Lai (2000) demonstrate that while monetary expansion leads to higher inflation and a lower money balances-capital ratio, it does not affect the marginal return on capital. This means that money is growth-rate superneutral when endogenous fertility is absent.[5] However, when endogenous fertility is present, even in the money-in-the-utility-function framework with the representative agent, monetary expansion can negatively affect the growth rate through changes in fertility. Moreover, our results also contrast with the findings of Brock (1974) in the presence of endogenous leisure. Brock (1974) states that monetary growth does not influence capital accumulation because of the unchanged labor when leisure and real balances are separable in the utility function, while this paper shows that money is not growth-rate superneutral even if fertility and real balances are separable in the utility function.

3.2. Transitional dynamics of an anticipated rise in the money growth rate

This section then examines the transitional dynamics of an anticipated rise in the money growth rate on the consumption-capital ratio x, the real balances-capital ratio z, fertility, and the balanced growth rate.

From (15), the general solutions for x and z are as follows:

$$x=\tilde{x}(\mu)+\frac{\tilde{x}-\tilde{z}(\alpha\tilde{x}/\tilde{z}^2)}{\tilde{z}[-(\alpha/\tilde{z})+1]}B_1e^{\upsilon_1 t}+\frac{\alpha\tilde{x}/\tilde{z}-\tilde{z}(\alpha\tilde{x}/\tilde{z}^2)}{\tilde{z}[-(\alpha/\tilde{z})+1]}B_2e^{\upsilon_2 t}, \tag{18a}$$

$$z=\tilde{z}(\mu)+B_1e^{\upsilon_1 t}+B_2e^{\upsilon_2 t}, \tag{18b}$$

where B_1 and B_2 are undetermined coefficients.

[4] Using (15) yields $\partial\tilde{x}/\partial\mu=0$ and $\partial\tilde{z}/\partial\mu=-\tilde{x}\tilde{z}/\Delta_1<0$. Substituting the above results into (10a) then yields (17a).

[5] When endogenous fertility is absent, the condition $\beta=0$ holds. Substituting this condition into (10a), (17a), and (17b), we obtain that $n_z=0$, $\partial\tilde{n}/\partial\mu=0$, and $\partial\tilde{g}/\partial\mu=0$.

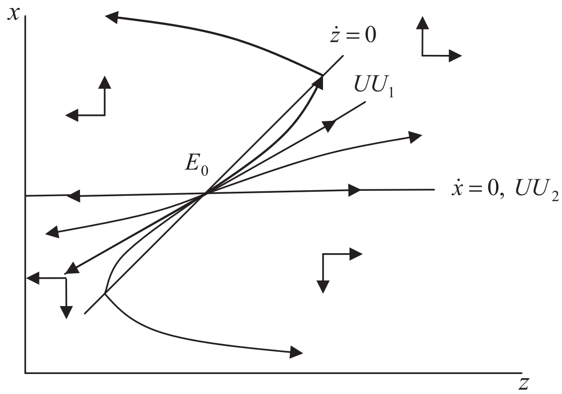

Fig. 1. Phase diagram.

Fig. 1 illustrates the dynamic characteristics of the system. Eq. (15) shows that the slopes of the loci $\dot{x} = 0$ and $\dot{z} = 0$ are:

$$\partial x/\partial z|_{\dot{x}=0} = 0, \tag{19a}$$

$$\partial x/\partial z|_{\dot{z}=0} = -\frac{\tilde{z}(\alpha\tilde{x}/\tilde{z}^2)}{\tilde{z}[-(\alpha/\tilde{z})+1]} > 0. \tag{19b}$$

The arrows in Fig. 1 indicate that the unstable arm UU_1 and the unstable branch UU_2 are associated with $B_2 = 0$ and $B_1 = 0$ in (18a) and (18b), respectively. UU_1 is flatter than the $\dot{z} = 0$ locus.[6] All other unstable trajectories in Fig. 1 correspond to the values with $B_1 \neq 0$ and $B_2 \neq 0$ in (18a) and (18b).

Figs. 2a–2c illustrate the transitional dynamics of an anticipated rise in the money growth rate on macroeconomic variables. Note the following points. First, the *nn* schedule in Fig. 2b depicts the instantaneous decision of fertility in (10a), given the consumption-capital ratio *x*, while the *gg* schedule in Fig. 2c depicts the economic growth rate exhibited in (13). Second, in Fig. 2a we consider an economy initially with a steady state E_0, which corresponds to the initial consumption-capital ratio x_0 and the initial real balances-capital ratio z_0. Corresponding to Figs. 2a–2c describe the initial fertility n_0 and the initial economic growth rate g_0 in association with the initial money growth rate μ_0. Third, the monetary authority announces that it will increase the money growth rate from μ_0 to μ_1 at time $t = T$. When an anticipated permanent increase in the money growth rate occurs, $\dot{z} = 0(\mu_0)$ shifts leftwards to $\dot{z} = 0(\mu_1)$. The new steady state is therefore E_1, and corresponds to the consumption-capital ratio x_0, the real balances-capital ratio z_1, the fertility n_1, the economic growth rate g_1, and the new money growth rate μ_1. Fourth, 0^+ represents the instant after the announcement, and T^- and T^+ respectively denote the instants before and after the policy switch.

According to Figs. 2a–2c, the announcement that the monetary authority will increase the money growth rate at a future date *T* induces the agent to expect a higher inflation rate in the future. At the instant 0^+, this expectation decreases the real balances-capital ratio *z* from z_0 to z_{0^+}, but does not influence the consumption-capital ratio *x* in Fig. 2a, in which the economy jumps from E_0 to E_{0^+}. This decline in the real balances-capital ratio *z* decreases fertility costs, thus increasing fertility from n_0 to n_{0^+} in Fig. 2b. This rise in fertility further slows down capital accumulation and thus depresses the economic growth rate from g_0 to g_{0^+} in Fig. 2c. During the transitional process, the real balances-capital ratio *z* continues to fall but the consumption-capital ratio *x* remains intact in Fig. 2a. At the same time, fertility rises in Fig. 2b, but the economic growth rate decreases in Fig. 2c. Finally, when the money growth rate rises to μ_1 at time T^+, the economy is at the new steady state E_1, which corresponds to the consumption-capital ratio x_0, the real balances-capital ratio z_1, the fertility n_1, and the economic growth rate g_1.[7]

These results run in sharp contrast to the findings of Chang and Lai (2000). Chang and Lai show that the intertemporal elasticity of substitution determines whether or not money is superneutral during the transition process in response to

[6] Given two characteristic roots, from (18a) and (18b), we have:

$$\partial x/\partial z|_{UU_1} = (\tilde{x} - \alpha\tilde{x}/\tilde{z})/\tilde{z}[-(\alpha/\tilde{z})+1] > 0,$$

$$\partial x/\partial z|_{UU_2} = 0,$$

$$\partial x/\partial z|_{\dot{z}=0} - \partial x/\partial z|_{UU_1} = -\tilde{x}/\tilde{z}[-(\alpha/\tilde{z})+1] > 0.$$

[7] From the instantaneous decision of fertility (10a) and the economic growth rate (13), we obtain: $\dot{n} = n_x\dot{x} + n_z\dot{z} = n_z\dot{z}$ and $\dot{g} = -(1+\bar{q})\dot{n}$, as *x* remains intact at the steady-state value x_0 during the whole of the adjustment process in Fig. 2a.

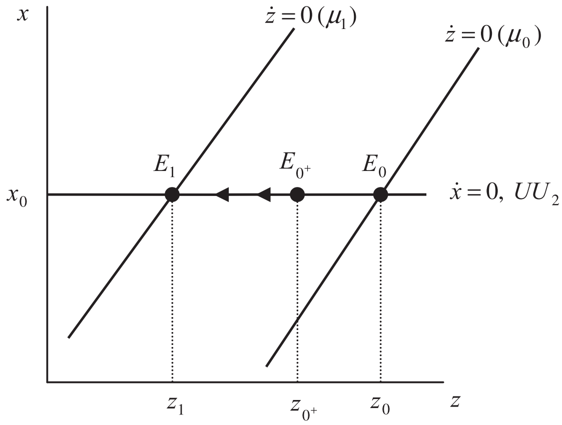

Fig. 2a. Transition of x and z.

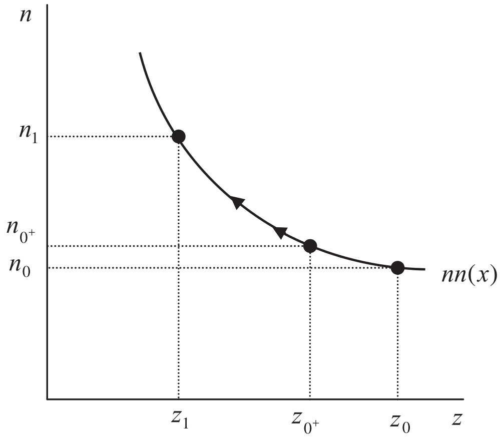

Fig. 2b. Transition of z and n.

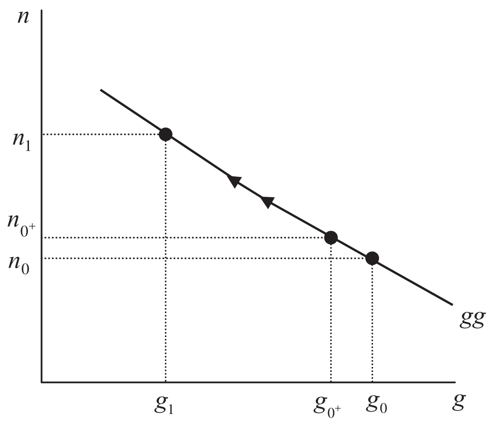

Fig. 2c. Transition of n and g.

an anticipated rise in the money growth rate. Specifically, when the intertemporal elasticity of substitution equals unity, the economy does not exhibit the transitional dynamics of the economic growth rate. By contrast, we show that even if the intertemporal elasticity of substitution equals unity, money is non-superneutral during the transitional process. This is because an anticipated monetary expansion influences the dynamics of the economic growth rate via changes in fertility. Accordingly, this study makes the following proposition:

Proposition 2. *When endogenous fertility is present, an anticipated increase in the money growth rate generates the transitional dynamics of fertility and the economic growth rate, even if the intertemporal elasticity of substitution equals unity.*

3.3. Optimal monetary policy

Friedman (1969) is a pioneer in exploring the topic of the optimum quantity of money supply and proposes an appropriate policy suggestion to the government. It is a worthwhile task for us to examine how endogenous fertility will govern the welfare effects of monetary policy and the optimal monetary policy.

Along the balanced growth path, we obtain that $c_t = c_0 e^{\tilde{g}t}$, $m_t = m_0 e^{\tilde{g}t}$, $c_0 = \tilde{x}k_0$, and $m_0 = \tilde{z}k_0$, where c_0 is the initial level of consumption, m_0 is the initial level of real balances, and k_0 is the initial level of capital stock. Thus, from (1) the social welfare function is given by:

$$W = \int_0^\infty [\ln c_0 + \alpha \ln m_0 + (1+\alpha)\tilde{g}t + \beta \ln \tilde{n}] e^{-\rho t} dt, \tag{20}$$

where $c_0 = \rho k_0$ and $m_0 = [\alpha c_0/(\mu + \rho)]$.[8] With (20), (17a), and (17b), the welfare effects of monetary growth can be derived:

$$\begin{aligned} \partial W/\partial \mu &= [(1/c_0)(\partial c_0/\partial \mu)/\rho] + [(\alpha/m_0)(\partial m_0/\partial \mu)/\rho] + [(1+\alpha)(\partial \tilde{g}/\partial \mu)/\rho^2] + [\beta(\partial \tilde{n}/\partial \mu)/\tilde{n}\rho], \\ &= [(\alpha x/z)D_a/\rho] + (D_b/\rho) \gtrless 0, \end{aligned} \tag{21a}$$

where $D_a = -\alpha/(\mu+\rho)^2 < 0$ and $D_b = \rho^{-1}\tilde{n}^{-1}[-(1+\alpha)(1+\bar{q})\tilde{n} + \beta\rho] \cdot (\partial \tilde{n}/\partial \mu)$. Based on (21a), we can establish:

Proposition 3. *In an AK growth model where money is in the utility function,*

(i) *social welfare falls with the money growth rate in the absence of endogenous fertility;*
(ii) *social welfare could rise or fall with the money growth rate in the presence of endogenous fertility.*

By imposing $\partial W/\partial \mu = 0$ in (21a), we can easily obtain the *optimal* nominal interest rate[9]:

$$R^* = (\alpha x/z)^* = -D_b/D_a \geqslant 0. \tag{21b}$$

Moreover, (8), (11), (12), (13), (21b), and the relationship $\tilde{g} = \dot{k}/k = \dot{c}/c = \dot{m}/m$ yield the following optimal money growth rate μ^*:

$$\mu^* = -D_b/D_a - \rho. \tag{21c}$$

Eqs. (21b) and (21c) show that both the optimal nominal interest rate R^* and the optimal money growth rate μ^* crucially depend on whether endogenous fertility is present.

In the absence of endogenous fertility, (21b) shows that the government should set the optimal nominal interest rate to zero, $R^* = 0$.[10,11] This is because the social cost of producing money is nil, while the private cost of holding money (the nominal interest rate) is greater than zero. The divergence between the private cost of holding money and the social cost of holding money leads to social inefficiency. To achieve the social optimum, the government should follow the Friedman (1969) rule and eliminate the private cost of holding money by setting a zero nominal interest rate, corresponding to a zero inflation tax. As $R^* = 0$, the return on capital (i.e., the real interest rate r) is equal to that on money ($-\pi$).

By contrast, it follows from (21b) that in the presence of endogenous fertility the optimal nominal interest rate R^* does not equal zero. Compared to the previous case where the fertility rate is fixed, in the presence of variable fertility rates real money balances give rise to an additional effect, namely, the wealth-narrowing effect, as emphasized above, increasing the costs of higher fertility. This in turn leads to lower fertility in the private economy and, accordingly, to a higher return on

[8] Eqs. (8), (12), and (13), and the relationships $\tilde{g} = \dot{k}/k = \dot{c}/c = \dot{m}/m$, $c_0 = \tilde{x}k_0$, and $m_0 = \tilde{z}k_0$ show that:

$$\dot{k}/k = A - (1+\bar{q})\tilde{n} - (c_0/k_0) = \tilde{g},$$

$$\dot{c}/c = A - \tilde{n} - \bar{q}\tilde{n} - \rho = \tilde{g},$$

$$\dot{m}/m = \mu - [\alpha(c_0/k_0)/(m_0/k_0)] + A - \bar{q}\tilde{n} - \tilde{n} = \tilde{g}.$$

Thus, $c_0 = \rho k_0$ and $m_0 = [\alpha c_0/(\mu+\rho)]$.

[9] Since the optimal monetary policy is a second-best policy, in which the government considers the reaction of the agent when setting the optimal monetary policy, the definition of the optimal nominal interest rate is the same as the one in (11).

[10] As indicated in Footnote 4, when endogenous fertility is absent, we have $\partial \tilde{n}/\mu = 0$. Substituting $\partial \tilde{n}/\mu = 0$ into (21b) shows that $R^* = 0$.

[11] The government will take steps to lower the money growth rate only when the nominal interest rate equals zero, which corresponds to the return on real balances equaling the return on capital. In addition, (9) and (10a) and $\dot{x}/x = \dot{z}/z = 0$ show that $x = \rho$ and $z = \alpha\rho/(\mu+\rho)$. The optimal money growth rate $\mu^* = -\rho$ is a corner solution that allows z to be a positive value.

capital in the private economy. In order to achieve the social optimum, the government should account for the additional wealth-narrowing effect and decrease the households' money holding. Thus, a *positive* nominal interest rate is optimal for the society to tax "money" and thereby to encourage capital accumulation. As a consequence, the Friedman rule is no longer optimal. We summarize the welfare results in the following proposition:

Proposition 4. *In an AK growth model with a money in the utility function,*

(i) *the welfare-maximizing monetary policy follows Friedman's rule in the absence of endogenous fertility;*
(ii) *the Friedman rule is not a welfare-maximizing monetary policy in the presence of endogenous fertility.*

4. Discussions

Several assumptions in Sections 2 and 3 are debatable. Therefore, in this section we will relax them and provide extensive discussions, accordingly.

4.1. Interdependence between consumption and real money balances

In the previous section, the utility function is log-additive separable, implying that consumption and real money balances are independent. To shed light on the interdependence between consumption and real balances, in this subsection we adopt a more generalized utility function by modifying (1) as follows:

$$U = \int_0^\infty \left\{ \frac{\left\{ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} n^\beta \right\}^{1-\sigma} - 1}{1-\sigma} \right\} e^{-\rho t} dt, \ \varepsilon > 0, \ 1 > \gamma > 0, \ \sigma > 0, \ \beta > 0, \tag{22}$$

where γ is the share parameter of consumption, σ is the coefficient of relative risk aversion, and β is a preference parameter measuring the desire for fertility. The parameter ε is the elasticity of substitution between consumption and real balances. It is used to measure the degree of complementarity/substitution between consumption and real balances in the utility function; specifically, the smaller the elasticity of substitution ε is, the stronger the complementarity is. It is easy to recover the utility function in Section 2 if we set $\varepsilon = 1$ and $\sigma = 1$.

The household's optimization problem is to maximize the lifetime utility function (22) subject to the same budget constraints, as reported in (2) and (3). By following the same solution procedure in Section 2, we have the first-order conditions as follows:

$$\gamma \left\{ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} n^\beta \right\}^{1-\sigma} \Big/ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right] c^{\frac{1}{\varepsilon}} = \lambda, \tag{23a}$$

$$\beta \left\{ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} n^\beta \right\}^{1-\sigma} \Big/ n = \lambda[(m+k) + \bar{q}k], \tag{23b}$$

$$\left\{ (1-\gamma) \left\{ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} n^\beta \right\}^{1-\sigma} \Big/ \left[\gamma c^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) m^{\frac{\varepsilon-1}{\varepsilon}} \right] m^{\frac{1}{\varepsilon}} \right\} - \lambda(\pi + n) = -\dot{\lambda} + \lambda\rho, \tag{23c}$$

$$\lambda(A - n - \bar{q}n) = -\dot{\lambda} + \lambda\rho. \tag{23d}$$

At the steady-state growth equilibrium, the economy is characterized by $\dot{x} = \dot{z} = 0$, with $x = \hat{x}$ and $z = \hat{z}$ being the steady-state ratios of consumption-capital and money-capital, respectively. Based on this modification, we re-derive the effects of monetary expansion on fertility, inflation, and growth, respectively:

$$\partial\hat{n}/\partial\mu = -\hat{x}\hat{z}\varepsilon\sigma\Lambda/\widehat{\Delta}_1\widehat{\Delta}_2 \gtrless 0, \tag{24a}$$

$$\partial\hat{\pi}/\partial\mu = 1 + [(1+\bar{q}-\sigma)/\sigma](\partial\hat{n}/\partial\mu) \gtrless 0, \tag{24b}$$

$$\partial\hat{g}/\partial\mu = \hat{x}\hat{z}\varepsilon(1+\bar{q})\Lambda/\widehat{\Delta}_1\widehat{\Delta}_2 \lesseqgtr 0. \tag{24c}$$

where $\Lambda = \frac{\beta\left[(1-\gamma)(\varepsilon-1)\hat{x}^{\frac{1}{\varepsilon}}\hat{z}^{-\frac{1}{\varepsilon}}(1+\bar{q}) - (1-\gamma)\hat{x}^{\frac{1}{\varepsilon}}\hat{z}^{\frac{\varepsilon-1}{\varepsilon}} - \varepsilon\gamma\hat{x}\right]}{\gamma\varepsilon(1+\bar{q}+\hat{z})^2}$ and $\widehat{\Delta}_1 = \frac{1}{\left[\gamma\hat{x}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\hat{z}^{\frac{\varepsilon-1}{\varepsilon}}\right]}\left[\varepsilon(\sigma - \beta(1-\sigma))\gamma\hat{x}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta(1-\sigma))(1-\gamma)\hat{z}^{\frac{\varepsilon-1}{\varepsilon}}\right] > 0$. To meet the stability condition, we further assume that $\widehat{\Delta}_2 = \hat{x}\{[(\mu+\hat{x})[\sigma\hat{x}(1+\bar{q}+\hat{z}) - (1-\sigma)(1+\bar{q})^2\hat{n}]/(1+\bar{q}+\hat{z})\hat{x}] - \sigma\bar{q}n_z\hat{z}\varepsilon + (1+\bar{q}-\sigma)n_z\hat{z}\varepsilon\}/\widehat{\Delta}_1$. The results of (24a)–(24c) can be summarized as:

Proposition 5. *In the presence of interdependence between consumption and real money balances,*

(i) *if consumption and real balances are complements ($\varepsilon < 1$) or independent ($\varepsilon = 1$), monetary expansion increases fertility and inflation, but decreases economic growth;*
(ii) *if consumption and real balances are substitutes ($\varepsilon > 1$), monetary expansion has ambiguous effects on fertility, inflation, and growth.*

In the presence of interdependence between consumption and real money balances, monetary growth gives rise to two effects – the wealth-narrowing effect and the preference effect – on the fertility and balanced-growth rates. The wealth-narrowing effect, as noted previously, indicates that when monetary expansion raises the inflation rate and lowers real money balances, the costs of child-raising decrease in response. This, on the one hand, increases the fertility rate and, on the other hand, decreases the balanced-growth rate. Nonetheless, the preference effect refers to a mixed impact on fertility and growth. If consumption and real balances are complements ($\varepsilon < 1$), a decrease in the real money balances is associated with a lower level of the marginal utility of consumption, leading the households to favor child-raising n over consumption c. This further reinforces the positive effect on fertility and the negative effect on economic growth. By contrast, if consumption and real balances are substitutes ($\varepsilon > 1$), real money balances turn out to be negatively related to the marginal utility of consumption. Thus, the decrease in the real money balances leads households to favor consumption c over child-raising n. It turns out that the preference effect contradicts the wealth-narrowing effect, resulting in a negative effect on fertility and a positive effect on economic growth. Under such a situation, in a way that is different from Proposition 1, monetary expansion may stimulate, rather than depress, the balanced-growth rate.[12]

A corollary to Proposition 5 is that, in response to a nominal demand shock (i.e., monetary growth μ), (i) the relationship between inflation and economic growth is non-monotonic, (ii) the relationship between fertility and inflation is also non-monotonic, and (iii) the relationship between fertility and growth is negative. While the counter-cyclical $\hat{g}$–$\hat{n}$ relationship is standard (which is consistent with a neo-Malthusian relationship), the relationships of $\hat{g}$–$\hat{\pi}$ and $\hat{n}$–$\hat{\pi}$ are novel. While conventional macro-models predict a negative relationship between inflation and growth, recent studies, e.g., Crosby and Otto (2000) and Ericsson et al. (2000) have found that there is no insignificantly negative inflation-growth relationship. Ahmed and Rogers (2000) even show that the inflation-growth relationship could be positive. Proposition 5 provides a theoretical explanation for the diversity in the inflation-growth relationship in the empirical literature. Besides, the mixed relationship between fertility and inflation is also consistent with the practical evidence. While there is no formal empirical observation for the inflation-fertility relationship, based on 1973–1987 data on the inflation rate and the growth rate of population obtained from the World Bank, there seems to be no obvious correlation between the inflation rate and the growth rate of population.

In the existing monetary literature without fertility considerations, Levine and Renelt (1992) quantitatively show that a 10% increase in the money growth rate results in a 0.02–0.04% reduction in the economic growth rate. Gomme's (1993) numerical analysis reveals that a 10% increase in the money growth rate results in a 0.06% reduction in the economic growth rate. Obviously, these existing studies have found that the growth effect of monetary expansion is very limited. One may therefore be interested in performing a simple numerical analysis in the model with endogenous fertility, to examine whether the effect of money growth on the balanced-growth rate could be substantially large.[13] Specifically, we attempt to examine how the magnitude of the growth effect of monetary policy reacts to the elasticity of substitution between consumption and real money balances ε, the parameter related to the cost of child-raising $\bar{q}$, and the fertility preference parameter β.

We first calibrate the benchmark parameter values and in turn quantify the growth effect of monetary expansion. We set the share parameter $\gamma = 0.95$, as in Holman (1998), and set the elasticity of substitution between consumption and real balances $\varepsilon = 0.5$, as in Hoffman et al. (1995). The coefficient of relative risk aversion is set as $\sigma = 2$ and the time preference rate is set as $\rho = 0.04$, which are commonly-used parameter values in the literature. We set the money growth rate as $\mu = 0.06$, which is within the plausible range of the actual data and in line with the related studies, e.g., Jonsson, 2007. With these parameters, we can calibrate $A = 0.1052$, $\bar{q} = 1.5263$, and $\beta = 0.3964$ in the determinate equilibrium of the steady state such that the balanced-growth rate $g = 0.02$, the inflation rate $\pi = 0.03$ (which are in line with the data for the US, see Benk et al., 2010), $n = 0.01$ (which is approximately equal to the average fertility rate for the US during 2001–2009), $c/y = 0.57$ (as in Jonsson, 2007), and $m/y = 0.38$ (which lies in the reasonable range of 0.166–0.584, see, for example, Heer and Süssmuth, 2007; Heer et al., 2011; Gillman and Kejak, 2011; Martin, 2011).

Based on the parameterization above, we establish:

Result 1. In the presence of interdependence between consumption and real money balances,

(i) if $\varepsilon < (>) 1.18$, monetary expansion has a negative (positive) effect on economic growth. With particular emphasis, as $\varepsilon = 0.01$, then a 10% increase in the money growth rate results in a 4.6% reduction in the long-run growth rate;

[12] This ambiguous effect of monetary expansion was pointed out to us by an anonymous referee, to whom we are grateful.
[13] We are grateful to an anonymous referee for bringing this interesting issue to our attention.

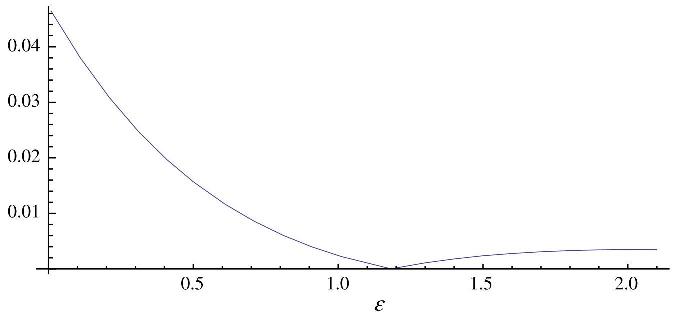

Fig. 3a. The change rate of the growth effect of monetary growth (under variable values of ε).

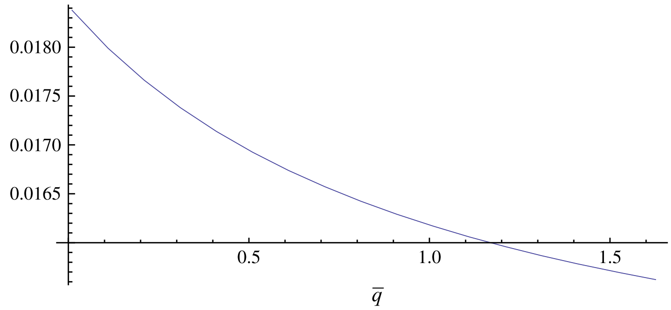

Fig. 3b. The change rate of the growth effect of monetary growth (under variable values of $\bar{q}$).

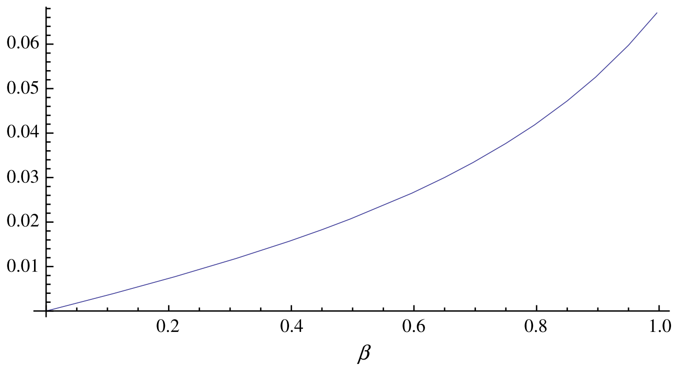

Fig. 3c. The change rate of the growth effect of monetary growth (under variable values of β).

(ii) given the benchmark parameter of $\varepsilon = 0.5$, the higher that the cost parameter of child-raising $\bar{q}$ is, the smaller is the growth effect of monetary policy. Specifically, if $\bar{q} = 0.01$, then a 10% increase in the money growth rate results in a 1.8% reduction in the long-run growth rate;

(iii) given the benchmark parameter of $\varepsilon = 0.5$, the stronger that the fertility preference β is, the larger is the growth effect of monetary policy. Specifically, if $\beta = 0.5$, then a 10% increase in the money growth rate results in a 2.1% reduction in the long-run growth rate.

The relationship between the growth effects of monetary policy and ε, $\bar{q}$, and β are shown in Figs. 3a–3c, respectively. Result 1 clearly shows that when we take an endogenous fertility rate into account, (i) monetary policy could have a substantially large effect on economic growth, compared to the results of existing studies, say, Levine and Renelt (1992) and Gomme (1993), and (ii) by focusing on the benchmark case where $\varepsilon = 0.5$, the growth effect of monetary policy decreases with the elasticity of substitution between consumption and real balances ε and the cost of child-raising $\bar{q}$, but increases with the fertility preference β. The intuition behind this result is straightforward. When the elasticity of substitution between consumption and real balances ε is smaller (consumption and real balances are complements), the preference effect, as shown

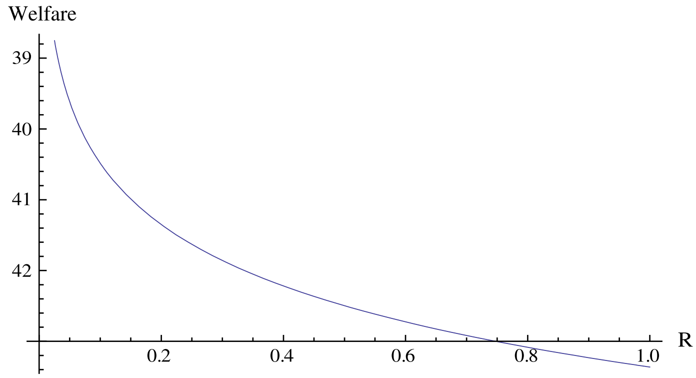

Fig. 4a. The optimal nominal interest rate in the absence of endogenous fertility.

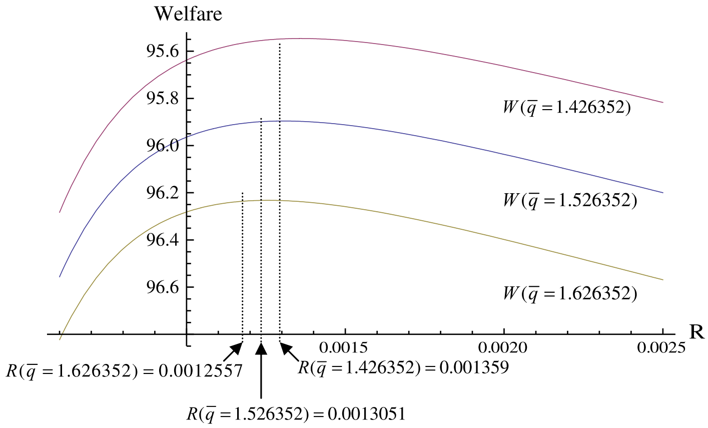

Fig. 4b. The optimal nominal interest rate and $\bar{q}$.

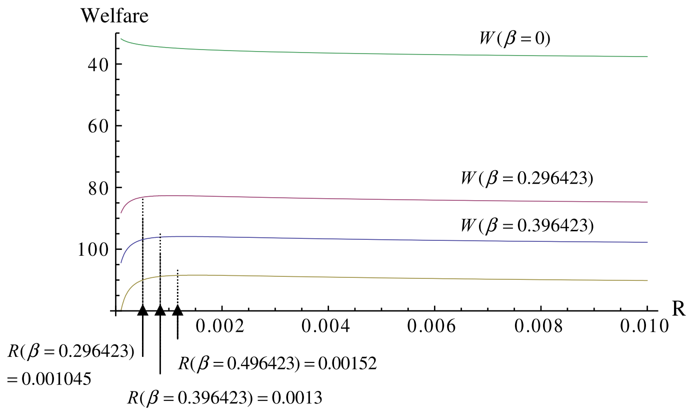

Fig. 4c. The optimal nominal interest rate and β.

in Proposition 5, reinforces the wealth-narrowing effect, resulting in a larger impact in terms of the reduction in economic growth. Therefore, the effect of monetary expansion on growth can be substantial. In addition, given $\varepsilon = 0.5$, either a stronger fertility preference β or a lower cost of child-raising $\bar{q}$ will amplify the effects stemming from the endogenous fertility and, as a result, the growth effect of monetary expansion will turn out to be more pronounced. To sum up, due to endogenous fertility, monetary policy could give rise to a substantially large effect on economic growth.

4.2. Friedman's rule and endogenous fertility: a numerical exercise

In this subsection we will engage in a simple numerical exercise in order to gain a better understanding of Propositions 3 and 4. This numerical analysis is undertaken under the parameter values in Section 4.1. Nevertheless, here we assume that consumption and real money balances are independent ($\varepsilon = 1$) in order to make our point more striking and specify a unitary elasticity of intertemporal substitution ($\sigma = 1$) for simplicity. This specification recovers that of the benchmark model in Sections 2 and 3. When $\beta = 0$, we can also recover the case where endogenous fertility is absent.

Fig. 4a first shows that, in the absence of the endogenous fertility, social welfare monotonically decreases with the nominal interest rate, implying that the optimal level of the nominal interest rate R^* is zero.

When endogenous fertility is present, Figs. 4b and 4c indicate that the optimal nominal interest rate R^* does not follow Friedman's rule and is no longer zero. To summarize these results, we have:

Result 2. In the presence of endogenous fertility, the optimal nominal interest rate is decreasing in the cost of child-raising $\bar{q}$, but increasing in the fertility preference β.

Proposition 4 has noted that by accounting for the wealth-narrowing effect of fertility, the government should tax money in order to discourage the households from holding money and encourage them to accumulate capital. Thus, a *positive* nominal interest rate is optimal for the society. As is evident, under the situation with either a lower cost of child-raising $\bar{q}$ or a stronger fertility preference β, the wealth-narrowing effect of endogenous fertility becomes more substantial. Thus, the government should levy more tax on the holding of money balances. Consequently, the optimal nominal interest rate R^* decreases with $\bar{q}$ and increases with β, as shown in Figs. 4b and 4c.

5. Conclusion

This paper develops a monetary endogenous growth model with endogenous fertility and uses it to systematically examine the positive and normative implications of monetary policy. In the positive analysis, both the long-run steady-state growth and short-run transition effects of money growth are uncovered. In the normative analysis, we not only analytically examine the validity of Friedman's rule, but also numerically investigate the optimal nominal interest rate and its relationship with fertility.

It has been shown that two distinct effects – the wealth-narrowing effect and the preference effect – govern the impacts of monetary policy on fertility and economic growth. Along a balanced growth path, monetary growth increases the fertility rate and reduces the growth rate if consumption and real balances are complements or independent. By contrast, monetary expansion may decrease the fertility rate and increase the growth rate if consumption and real balances are substitutes. Even in the money-in-the-utility-function framework without a labor-leisure choice, the superneutrality of money does not hold. By running in sharp contrast to the findings of Chang and Lai (2000), an anticipated increase in the money growth rate generates the transitional dynamics of fertility and the economic growth rate even if the intertemporal elasticity of substitution equals unity, provided that endogenous fertility is taken into account. Our normative analysis has indicated that in the presence of endogenous fertility the Friedman rule is no longer optimal. Due to the wealth-narrowing effect stemming from the variable fertility rates, the government should tax the holding of money by setting a positive nominal interest rate. Since either a lower cost of child-raising or a stronger fertility preference reinforces the wealth-narrowing effect, a higher nominal interest rate is desired for the society in order to achieve the social optimum.

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