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A CLASSICAL TAX-SUBSIDY PROBLEM[1]

GERARD DEBREU

SUMMARY

Of Section 1: In the economic system l commodities (given quantities of which are initially available) are transformed into each other by n production-units (the technological knowledge of which is limited) and consumed by m consumption-units (the preferences of which are represented by m satisfaction functions).

Of Section 2: Given a set of values $(s_1, \cdots, s_i, \cdots, s_m)$ of the individual satisfactions, the economic efficiency of this situation is defined as follows: the quantities of all the available resources are multiplied by a number such that using these new resources and the same technological knowledge as before it is still possible to achieve for the i^{th} individual $(i = 1, \cdots, m)$ a satisfaction at least equal to s_i. The smallest of the numbers satisfying this condition is ρ, the coefficient of resource utilization of the economy in this situation[2] [**3**]; ρ equals one for a Pareto optimal situation, is smaller than one for a nonoptimal situation. In the last case the loss of efficiency is $1 - \rho$; a more interesting quantity (immediately derived from $1 - \rho$) is the money value of the resources which might be thrown away while still permitting the achievement of the same individual satisfactions. This latter quantity will be referred to as *the economic loss* associated with the situation $(s_1, \cdots, s_i, \cdots, s_m)$.

This definition of the loss has been critically compared in [**3**] with other definitions. We will only recall here that these took as a reference an *arbitrarily selected* optimal state (which was, in the problems of Sections 4–5, the initial state); moreover they could associate with two states of the system, where *all* satisfactions were the *same*, two different values of the economic loss.[3] The one exception to which these criticisms do not apply is the definition given by M. Allais [**1**] p. 638, but the latter rests on the *arbitrary choice* of a particular commodity.

The tastes, the technology, and the available resources are fixed in this paper and ρ is thus a function $\rho(s_1, \cdots, s_m)$ of the individual levels of satisfaction.

[1] This paper (based on Cowles Commission Discussion Paper, Economics No. 2020, July, 1951) was presented at the Minneapolis Meeting of the Econometric Society, September, 1951. It was prepared as part of a research project on the theory of allocation of resources conducted by the Cowles Commission for Research in Economics under contract with the RAND Corporation. It will be reprinted as Cowles Commission Paper, New Series, No. 80.

The conception of this study goes back to my reading correspondence of 1946 between M. Allais and H. Hotelling. I am very grateful to both of them for having made it available to me. I have also benefited greatly from the comments of M. Boiteux, T. C. Koopmans, and other Cowles Commission staff members and guests.

[2] The number ρ is clearly independent of the arbitrary monotonic increasing transformations affecting the satisfaction functions.

[3] The other definitions could also yield a *positive* loss for an optimal state and/or a *negative* loss for a nonoptimal state.

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Of Section 3: The first and second differentials of this function play an essential role later on; they are calculated in this section.

Of Section 4: We consider now as an initial state of the economic system a Pareto optimal situation observed in a competitive market economy. In the final state, which is in the neighborhood of the initial state, all consumption-units maximize their satisfactions according to a certain price system but the behavior of production units is entirely unspecified. Under these conditions one obtains from the results of Section 3 the general approximation formula (14) for the economic loss.

Of Section 5: As an application we consider the case where a system of indirect taxes and subsidies is introduced in a Pareto optimal state of the economy and we obtain the approximation formula (17) for the resulting economic loss.

The problem of the evaluation of this loss was first considered by J. Dupuit. His work stimulated a large number of further studies which have been reviewed by H. Hotelling in [**5**]. H. Hotelling himself bases his study on the assumption that in the economic change which takes place the first differential of every satisfaction vanishes, in other words that there are no (first order) income redistribution effects. Formula (17) then reduces to the much more simple formula (18); the choice among the various definitions of the loss also becomes irrelevant for these definitions practically all lead to the same result in this particular case. Only in exceptional circumstances, however, will the income redistribution effects of a system of indirect taxes and subsidies be of the second order.

The recent article [**2**] of M. Boiteux starts from a definition of the loss to which several of the criticisms listed at the end of the summary of section 2 apply.

1. THE ECONOMIC SYSTEM

The activity of the economic system extends over a finite number of consecutive time-intervals and takes place in a finite number of locations. A commodity is characterized by its physical properties, the time-interval during which it is available and the place at which it is available. There are altogether l commodities (goods or services,[4] direct or indirect, etc.).

A consumption-unit is characterized by a subscript i ($i = 1, \cdots, m$). Its activity is represented by a consumption vector x_i of the l-dimensional commodity space R_l; the components of x_i are quantities of commodities actually consumed or negatives of quantities of commodities produced (for example, different kinds of labor). The tastes of the consumption-unit are represented by a satisfaction function $s_i(x_i)$, defined but for an arbitrary monotonically increasing transformation. The *total* consumption-vector is $x = \sum_i x_i$.

A production-unit is characterized by a subscript j ($j = 1, \cdots, n$). Its activity is represented by an input-vector y_j of R_l; the components of y_j are inputs (net quantities of commodities consumed) or negatives of outputs (net quantities of commodities produced). The limitation of technological knowledge constrains y_j to satisfy the equation $e_j(y_j) = 0$ of the production surface. (We are not

[4] Including transportation services.

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concerned in this paper with technological inefficiencies, i.e., with points that are above the production surfaces). The *total* input-vector is $y = \sum_j y_j$.

Finally, if z^0 is the utilizable physical resources vector (whose components are the available quantities of each commodity), the total *net* consumption of the whole economic system must equal z^0: $x + y = z^0$. (We are not concerned with free commodities for which the total net consumption would be smaller than the available quantity.)

In a given state of the economy some components of x_i are null (commodities neither consumed nor produced by the i^{th} consumption unit); throughout this paper we shall assume that, in the neighboring states of the economy considered, the set of null components of x_i is *the same* as in the initial state. This assumption is made for all $i = 1, \cdots, m$ and similarly for all y_j, $j = 1, \cdots, n$. $R_l^i(R_l^j)$ denotes the subspace of R_l corresponding to the nonzero components of $x_i(y_j)$; the projection of a vector such as p on $R_l^i(R_l^j)$ is denoted by $p^i(p^j)$; $x^i(y^j)$ stands for $x_i^i(y_j^j)$.

Functions are supposed to be continuous and differentiable whenever necessary.

2. THE COEFFICIENT OF RESOURCE UTILIZATION

An economic situation is defined by the values $(s_1, \cdots, s_m)$ of the m satisfactions. The efficiency of this situation is measured by ρ, the coefficient of resource utilization of the economic system **[3]**, the smallest number by which the quantities of all the available resources can be multiplied while still permitting for every consumption unit a satisfaction $s_i(x_i)$ at least equal to s_i. Thus, for an economic system whose technology is described by the n functions $e_j(y_j)$ and whose available resources vector is ρz^0, the situation $(s_1, \cdots, s_m)$ is optimal (in Pareto's sense). Therefore, under suitable convexity assumptions, there exists **[3]** an intrinsic price vector $\bar{p}$ to which consumption units and production units are adapted according to the familiar pattern (satisfaction maximization and profit maximization). In mathematical terms, there exist a vector $\bar{p}$, m vectors $\bar{x}_i$, n vectors $\bar{y}_j$, m numbers $\bar{\sigma}_i$ and n numbers $\bar{\epsilon}_j$ such that

$$(1) \qquad \begin{cases} s_i(\bar{x}^i) = s_i, & e_j(\bar{y}^j) = 0, \\[4pt] \dfrac{ds_i}{d\bar{x}^i} = \bar{\sigma}_i \bar{p}^i, & \dfrac{de_j}{d\bar{y}^j} = \bar{\epsilon}_j \bar{p}^j, \\[4pt] \sum_i \bar{x}_i + \sum_j \bar{y}_j = \rho z^0. \end{cases}$$

The first and the third equations hold for all i, the second and the fourth for all j. The same convention will be made for all equations of this form.

The tastes (the m functions $s_i(x_i)$), the technology (the n functions $e_j(y_j)$), and the available resources (z^0) are fixed throughout this paper. ρ is thus a function $\rho(s_1, \cdots, s_m)$ implicitly defined by (1).

The loss of efficiency associated with the situation $(s_1, \cdots, s_m)$ is $1 - \rho$.

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3. FIRST AND SECOND DIFFERENTIALS OF ρ

Let S^i be the hessian[5] matrix of $s_i(x^i)$, and similarly let E^j be that of $e_j(y^j)$. Differentiate[6] all equations of (1)

$$(2)\qquad \begin{cases} \dfrac{ds_i}{d\bar{x}^i}\cdot d\bar{x}^i = ds_i, \qquad \dfrac{de_j}{d\bar{y}^j}\cdot d\bar{y}^j = 0, \\ S^i d\bar{x}^i = \bar{\sigma}_i\, d\bar{p}^i + \bar{p}^i d\bar{\sigma}_i, \qquad E^j\, d\bar{y}^j = \bar{\epsilon}_j\, d\bar{p}^j + \bar{p}^j d\bar{\epsilon}_j, \\ \sum_i d\bar{x}_i + \sum_j d\bar{y}_j = z^0\, d\rho. \end{cases}$$

From the third and fourth equations of (1), designated (1_3) and (1_4), the first and second equations of (2), designated (2_1) and (2_2), become

$$(3)\qquad \bar{\sigma}_i(\bar{p}^i)'d\bar{x}^i = ds_i, \qquad \bar{\epsilon}_j(\bar{p}^j)'d\bar{y}^j = 0.$$

Premultiplication of (2_5) by $\bar{p}'$ yields

$$(4)\qquad \bar{p}'\sum_i d\bar{x}_i + \bar{p}'\sum_j d\bar{y}_j = \bar{p}'z^0 d\rho.$$

Since $(\bar{p}^i)'d\bar{x}^i = \bar{p}'d\bar{x}_i$ and $(\bar{p}^j)'d\bar{y}^j = \bar{p}'d\bar{y}_j$, (3) and (4) give

$$(5)\qquad \bar{p}'z^0\, d\rho = \sum_i \frac{ds_i}{\bar{\sigma}_i}.$$

The first differential of ρ is thus known. To obtain the second differential of ρ we differentiate (5) to obtain

$$(6)\qquad \bar{p}'z^0\, d^2\rho + d\bar{p}'z_0\, d\rho = \sum_i \left(\frac{d^2 s_i}{\bar{\sigma}_i} - \frac{ds_i\, d\bar{\sigma}_i}{\bar{\sigma}_i^2}\right).$$

However, $d\bar{p}$ and $d\bar{\sigma}_i$ are not known and the rest of the present section is devoted to the solution of this difficulty.

Rewrite (2_3) and (3_1) as follows:

$$\begin{cases} \dfrac{1}{\bar{\sigma}_i} S^i d\bar{x}^i - \bar{p}^i \dfrac{d\bar{\sigma}_i}{\bar{\sigma}_i} = d\bar{p}^i \\ (\bar{p}^i)'d\bar{x}^i = \dfrac{ds_i}{\bar{\sigma}_i}. \end{cases}$$

[5] $S^i = \left[\dfrac{\partial^2 s_i}{\partial x_h^i \partial x_k^i}\right]$ where x_h^i, x_k^i are coordinates of x^i.

[6] Primed letters indicate transposes of matrices and for the inner product of two (column) vectors, u, v we use indifferently the notations $u \cdot v$ and $u'v$. For a presentation of differentials see, for example, [7].

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If $\sum^i = \begin{bmatrix} \frac{1}{\bar{\sigma}_i} S^i & \bar{p}^i \\ (\bar{p}^i)' & 0 \end{bmatrix}$, the above system can be written

$$\sum^i \begin{bmatrix} d\bar{x}^i \\ -\frac{d\bar{\sigma}_i}{\bar{\sigma}_i} \end{bmatrix} = \begin{bmatrix} d\bar{p}^i \\ \frac{ds_i}{\bar{\sigma}_i} \end{bmatrix}, \text{ i.e., } \begin{bmatrix} d\bar{x}^i \\ -\frac{d\bar{\sigma}_i}{\bar{\sigma}_i} \end{bmatrix} = (\sum^i)^{-1} \begin{bmatrix} d\bar{p}^i \\ \frac{ds_i}{\bar{\sigma}_i} \end{bmatrix}. \tag{7}$$

$(\sum^i)^{-1}$ has the form $(\sum^i)^{-1} = \begin{bmatrix} X^i & \gamma^i \\ (\gamma^i)' & c^i \end{bmatrix}$ where γ^i is a column vector, c^i a number.[7] Therefore (7_2) is equivalent to

$$\begin{cases} d\bar{x}^i = X^i d\bar{p}^i + \gamma^i \dfrac{ds_i}{\bar{\sigma}_i} \\ -\dfrac{d\bar{\sigma}_i}{\bar{\sigma}_i} = (\gamma^i)' d\bar{p}^i + c^i \dfrac{ds_i}{\bar{\sigma}_i}. \end{cases} \tag{8}$$

Let X_i be the $l \cdot l$ matrix obtained by inserting in X^i rows and columns of zeros corresponding to the zero components of x_i. Let γ_i be the $l \cdot 1$ matrix obtained by inserting similarly zeros in γ^i. It is clear that in (8), $\bar{x}^i$, X^i, $\bar{p}^i$, γ^i can be replaced by $\bar{x}_i$, X_i, $\bar{p}$, γ_i.

We add then all the equations (8_1) denoting by $X = \sum_i X_i$ the aggregate consumption substitution matrix.

$$d\bar{x} = X d\bar{p} + \sum_i \gamma_i \frac{ds_i}{\bar{\sigma}_i}. \tag{9}$$

Since (2_3) and (3_1) are identical in form to (2_4) and (3_2) one obtains without any calculation.

$$d\bar{y} = Y\, d\bar{p} \tag{10}$$

where the aggregate production substitution matrix Y is derived from the E^j exactly as X is derived from the S^i.

Let finally $Z = X + Y$ be the aggregate substitution matrix; one obtains by addition of (9) and (10) (See (2_5)).

$$Z d\bar{p} = z^0 d\rho - \sum_i \gamma_i \frac{ds_i}{\bar{\sigma}_i}. \tag{11}$$

[7] The elements of $(\sum^i)^{-1}$ are familiar concepts: γ^i is the derivative $\partial x^i/\partial r$ of demanded quantities x^i with respect to income r when prices are constant; the elements of X^i are the classical substitution terms; the equations of Slutsky amount to the remark that the matrix X^i is symmetric, an immediate consequence of the fact that S^i is symmetric; c^i is the only term of $(\sum^i)^{-1}$ which depends on the arbitrary monotonically increasing transformation affecting $s_i(x^i)$.

The convexity assumptions [3] underlying this study imply that X^i, Y^j and therefore X, Y, Z are semi-definite negative matrices.

Strict convexity must in fact be assumed here so that matrices such as $(\sum^i)^{-1}$ may exist.

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All the terms of the right hand member are known; however the matrix Z is singular and $d\bar{p}$ is not uniquely determined (this was obvious *a priori* since $\bar{p}$ is determined but for a multiplication by a positive number). Still two distinct solutions $(d\bar{p})^1$ and $(d\bar{p})^2$ of (11) give the same value for $d^2\rho$ in (6) (The value of $-\frac{d\bar{\sigma}_i}{\bar{\sigma}_i}$ is obtained from (8_2)).

4. A GENERAL EXPRESSION OF THE ECONOMIC LOSS

We consider as an initial state of the system a Pareto optimal situation $(s_1^0, \cdots, s_m^0)$ observed in a competitive market economy (the superscript 0 shall refer to the initial point); equations (1) are therefore satisfied by the actually observed values p^0, x_i^0, y_j^0, σ_i^0, ϵ_j^0 and by the value $\rho^0 = 1$ [**3**].

A new observed state of the system is characterized by the values of the observed quantities (denoted by non-barred letters). The new values $(s_1, \cdots, s_m)$ of the satisfactions determine, by means of equations (1), the values of the quantities (denoted by barred letters) characterizing a certain hypothetical state of the system. We must consider this hypothetical state in order to determine the value of ρ associated with the new observed state. At the initial point the values of barred and non-barred letters are equal.

If ρ is the value of the coefficient of resource utilization for the new observed state, the change $\Delta\rho = \rho - \rho^0$ is given by the development

$$\Delta\rho = d\rho + \tfrac{1}{2}d^2\rho + \cdots .$$

Since $\rho^0 = 1$ is the maximum [**3**] of the function $\rho(s_1, \cdots, s_m)$ under the constraints $e_j(y_j) = 0$ and $x + y = z^0$, $d\rho = 0$ at the initial point. An approximation formula for $\Delta\rho$, based on the lowest order term of the above development, is thus $\frac{1}{2}d^2\rho$, and the money value of the economic loss admits the approximation $-\frac{1}{2}p^0 \cdot z^0 d^2\rho$.

We assume that in every observed state of the economy, there is a price vector p according to which every consumption-unit maximizes its satisfaction. Then, as in (3), $\frac{ds_i}{\sigma_i} = p'dx_i$ which yields after differentiation and summation over i

$$dp'dx + p'd^2x = \sum_i \left(\frac{d^2s_i}{\sigma_i} - \frac{ds_i\, d\sigma_i}{\sigma_i^2}\right). \tag{12}$$

Subtract (12) from (6) (remembering that at the initial point $d\rho = 0$ and barred letters coincide with non-barred letters).

$$p^0 \cdot z^0 d^2\rho - [dp \cdot dx + p^0 \cdot d^2x] = \sum_i \frac{ds_i}{\sigma_i^0}\left(\frac{d\sigma_i}{\sigma_i^0} - \frac{d\bar{\sigma}_i}{\sigma_i^0}\right). \tag{13}$$

(8) holds for non-barred as well as for barred letters, so (8_2) gives

$$\sum_i \frac{ds_i}{\sigma_i^0}\left(\frac{d\sigma_i}{\sigma_i^0} - \frac{d\bar{\sigma}_i}{\sigma_i^0}\right) = \sum_i \frac{ds_i}{\sigma_i^0}\gamma_i'(d\bar{p} - dp).$$

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(8_1) gives in turn $\frac{ds_i}{\sigma_i^0}\gamma_i' = (dx_i - X_i dp)'$. Denoting $d\xi = dx - Xdp$, we finally obtain from (13)

$$(14) \qquad -\tfrac{1}{2}p^0\cdot z^0 d^2\rho = -\tfrac{1}{2}[dp\cdot dx + p^0\cdot d^2x + d\xi\cdot(d\bar{p} - dp)].$$

The right-hand member is the general expression of the economic loss that we sought. Every vector therein is an *observed* vector with the exception of $d\bar{p}$ which is obtained (see (11) and (8_1)) as a solution of the equation

$$Zd\bar{p} = -d\xi.$$

(We emphasized at the end of Section 4 that any solution may be chosen.)

$d\xi$ is the excess of dx, the actual variation of total consumption, over Xdp, the variation which would take place if consumption-units were confronted with the actual price change dp, while their satisfactions were all held constant. If one assumes, as Hotelling does in [**5**], that in the actual change the first differential of every satisfaction vanishes ($ds_i = 0$, i.e., $p\cdot dx_i = 0$ for all $i = 1, \cdots, m$),[8] then $d\xi = 0$, and (14) takes a notably more simple form, already given by Allais [**1**] p. 616.

$Zd\bar{p}$ is the variation in the net consumption of the whole economy corresponding to the price change $d\bar{p}$ when all satisfactions are held constant and all production-units maximize their profits on the basis of $\bar{p}$.

5. APPLICATION TO THE TAX-SUBSIDY CASE

The initial state of the economy is the same as in section 4[9] and a system of indirect taxes and subsidies is introduced.[10] Since in every observed state of the economy every consumption-unit is adapted to the price system p, we can apply formula (14) to the present particular case.

In a precise way, the amount t_h^+ is paid by every production-unit for every unit of the h^{th} commodity ($h = 1, \cdots, l$) which is an input and the amount t_h^- for every unit which is an output. t_h^+ or t_h^- is positive for a tax, negative for a subsidy.[11] The tax-subsidy system is thus represented by two vectors t^+ and t^- in R_l. Its net return is redistributed to consumption-units in a way which is left unspecified.

[8] In general it can only be said that $p\cdot dx = \sum_i p\cdot dx_i = 0$ since $d\rho = 0$.

[9] Thus there may exist direct taxes of the following kind: a given (positive or negative) amount θ_i is paid by the ith consumption-unit and $\sum_i \theta_i = 0$.

[10] We might also consider the economy as partitioned into nations and introduce further a system of tariffs (which is formally identical with a system of taxes on transportation services). Formula (17) would then give an approximation of the economic loss for the set of trading nations as a whole; however a tariff on a commodity is not, in general, a small enough fraction of the price for that approximation to be satisfactory.

[11] These taxes and subsidies might as well be given in the form of percentages of the prices.

It must also be remarked that proportional taxes paid by consumption units for the commodities that they buy (or sell like labor) are *equivalent* to the same taxes paid by production-units when they sell (or buy) those commodities.

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The production function of the j^{th} production-unit can be written in the form $e_j(y_j^+, y_j^-) = 0$ where the components of y_j^+ (y_j^-) coincide with those of y_j when they are positive (negative) and are null otherwise. Let p be the price system prevailing throughout the economy. The j^{th} production-unit behaves as if the prices of its inputs were actually $(p + t^+)^j$ and the prices of its outputs $(p - t^-)^j$; since it maximizes its profit on the basis of the price system p and the tax-subsidy system t^+, t^-, the relation

$$(p + t^+)^j \cdot d(y_j^+)^j + (p - t^-)^j \cdot d(y_j^-)^j = 0 \tag{15}$$

holds in every observed state of the economy. Denoting[12] $q^+ = \sum_j y_j^+$, $q^- = \sum_j y_j^-$ and summing the equations (15) over j we obtain

$$(p + t^+) \cdot dq^+ + (p - t^-) \cdot dq^- = 0.$$

Differentiating, and noting that at the initial point $t^+ = 0 = t^-$, we obtain

$$(dp + dt^+) \cdot dq^+ + (dp - dt^-) \cdot dq^- + p^0 \cdot d^2q^+ + p^0 d^2 q^- = 0.$$

Since $q^+ + q^- + x = z^0$,

$$dt^+ \cdot dq^+ - dt^- \cdot dq^- = dp \cdot dx + p^0 \cdot d^2 x. \tag{16}$$

(14) and (16) finally yield the approximation formula for the money value of the economic loss due to the tax-subsidy system:

$$-\tfrac{1}{2}[dt^+ \cdot dq^+ - dt^- \cdot dq^- + d\xi \cdot (d\bar{p} - dp)]. \tag{17}$$

Its interpretation is as follows:

Select any commodity, say the h^{th}; dt_h^+ is the amount of the tax (or subsidy) on a unit of the h^{th} commodity when it is an input; dq_h^+ is the variation of the *gross* input of the h^{th} commodity for the whole production sector due to the introduction of the tax-subsidy system. The products $dt_h^+ \times dq_h^+$ are formed and added for all commodities. One proceeds similarly to obtain $dt^- \cdot dq^-$ and so to obtain the first term

$$-\tfrac{1}{2}[dt^+ \cdot dq^+ - dt^- \cdot dq^-] \tag{18}$$

to which (17) reduces when $ds_i = 0$ for all $i = 1, \cdots, m$. The interpretation of all the elements of the second term

$$-\tfrac{1}{2} d\xi \cdot (d\bar{p} - dp) \tag{19}$$

has been given at the end of section 4.[13]

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[12] Note that q^+ (q^-) is different from y^+ (y^-) formed from y as y_j^+ (y_j^-) was formed from y_j.

[13] The net return of the tax-subsidy system is $q^+ \cdot dt^+ - q^- \cdot dt^-$.

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[14] A good survey of the literature up to 1938 has been made by H. Hotelling in [5]. To my knowledge, [2] is the only study published since then dealing with the specific problem of evaluating the economic loss associated with a system of indirect taxes and subsidies. However, a score of articles have studied other efficiency aspects of taxation during the same period (see for example the bibliography of M. Friedman [4]). The closely related work of M. Allais [1] must also be added.

References concerning Pareto optimal states and the general problem of definition of the economic loss will be found in [3].

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